Some modifiers of conditionals

0. Introduction

The purpose of this paper is to analyse in the framework of Boolean semantics the meaning contribution of various categorially polyvalent particles (CPPs) when they occur in conditional sentences (CSs). CPPs are functional expressions which can have as their possible arguments expressions of different grammatical categories. Grammatically such expressions are usually *modifiers*, that is functional expressions of the category C/C for various categories C.

The classical cases of such categorially polyvalent modifiers are items like *only, also* and *even* as shown in the following examples:

(1a) (Only/also/even Leo) danced on weekdays with Lea in the garden
(1b) Leo (only/also/even danced) on weekdays with Lea in the garden
(1c) Leo danced (only/also/even on weekdays) with Lea in the garden
(1d) Leo danced on weekdays (only/also/even with Lea) in the garden
(1e) Leo danced on weekdays with Lea (only/also/even in the garden)
(2) Leo danced on weekdays with Lea in the garden

Of course, as we will see in some detail, classical CPPs can also modify CSs. Various analyses of CSs modified by some classical CPPs have been proposed (Lycan 1991). We will see, and this is an empirical contribution of this paper, that there are many other categorially polyvalent CPPs which also can modify CSs. Furthemore, it will be shown that classical CPPs, at least *only* and *even*, are logically basic in the sense that many other "non-classical" CPPs can be obtained from classical ones by Boolean operations.

1. Other cases

In this section we present some other cases of categorially polyvalent modifiers. First notice the following examples:

(3a)Some teachers, in particular/especially Leo, think that ...
(3b)Yesterday he did many things, in particular he finished his paper.
(3c) He sings everywhere, in particular in his bathroom.
(4a) Leo will not come, let alone Lea.
(4b) Leo does not work on Saturdays, let alone on Sundays.
(4c) Leo does not smoke, let alone drink.

Surprisingly, *at least, at most* are also categorially polyvalent modifiers:

(5a) At least/at most Lea will pass the examination.
(5b)Lea sings at least/at most in the bathroom.
(5c)At least/at most five teachers were there.

Many FPs occur in CSs. The cases *only, also* and *even* are well-known (cf. *only if, also if* and empheven if . In (6c) we have a modification of an *if*-clause by *at least* and (6d) shows that such a modification by *at most* is impossible:

(6a)Lea will be happy, in particular if Leo calls.
(6b) Lea will not be happy if it rains, let alone if it snows.
(6c) Lea will call, at least if it rains.
(6d) *Lea will call, at most if it rains.

There are similarities and differences between various CPPs. *only, also* and *even*, *at least* and *at most* need not occur with additional lexical material when applying to a particular argument. This does not seem to be the case with particles like *especially* and *in particular*. Furthermore, there is a systematic semantic relationship between the additional lexical material and the argument of these particles suggesting that the explicitly required lexical material plays a role of an anaphora-antecedent like element:

(7a)*In particular/especially Leo will call.
(7b) Some students, in particular Leo, will call.
(7c) Some students and in particular Leo, will call.
(7d) *He sings in his office, in particular/especially in the bathroom.
(8a) He sings everywhere, in particular in his bathroom.
(8b) He sings in his office and in particular in his bathroom.
(8c) *He likes wine, in particular chocolate.
(8d) He likes wine, in particular champagne.
(8e) He likes wine and in particular chocolate.

In spite of such differences it is possible to analyse CPPs in an uniform way using algebraic tools of the Boolean semantics.

2.Boolean semantics:

Boolean semantics (Keenan and Faltz 1985): for any category C there is a corresponding denotational Boolean algebra D_C of possible denotations of expressions of category C. The algebra $D_{A/B}$ has as elements functions from D_B to D_A. D_C are atomic. Atoms of the algebra $D_{A/B}$ are determined by atoms and/or elements of the resulting algebra D_A.

We are interested in denotational algebras of modifiers. A modifier is a functional expression of category C/C for various choices of C. Modifiers of category C/C denote in the denotational algebra of restrictive functions $RESTR(C)$, which is a subset of the set of functions from D_C onto D_C. The set $RESTR(C)$ of restrictive functions $f_c \in D_{C/C}$, is the set of functions satisfying the condition $f_c(x) \leq x$, for any $x \in D_C$ (Keenan and Faltz 1985). The set of restrictive functions forms a Boolean algebra:

Prop 1: Let B be a Boolean algebra. Then the set of functions f from B onto B satisfying the condition $f(x) \leq x$ forms a Boolean algebra R_B with the Boolean operations of meet and join defined pointwise and where $0_{R_B} = 0_B$, $1_{R_B} = id_B$, $f'(x) = x \cap (f(x))'$.

Prop 1 shows how to form the restrictive Boolean algebra R_B from the algebra B. What is important here is the fact that the Boolean complement is relativised to the one element of the algebra which is just the identity function.

Restrictive algebras are also atomic:

Prop 2: If B is atomic so is R_B. For all $b \in B$ and all atoms α of B such that $\alpha \leq b$, functions $f_{b,\alpha}$ defined by $f_{b,\alpha}(x) = \alpha$ if $x = b$ and $f_{b,\alpha}(x) = 0_B$ if $x \neq b$ are the atoms.

There is an important sub-class $ABS(B)$ of restrictive functions (relative to a given Boolean algebra B): these are the so-called *absolute functions*. By definition $f \in ABS(B)$ iff for any $x \in B$, we have $f(x) = x \cap f(1_B)$. One can show that $ABS(B)$ is a sub-algebra of R_B.The atoms and co-atoms of $ABS(B)$ are indicated in:

Prop 3: If B is atomic so is $ABS(B)$. For all atoms α of B, functions f_α, defined by $f_\alpha(x) = \alpha \cap x$ are the atoms of $ABS(B)$. For all atoms α of B, functions f_α, defined by $f_\alpha(x) = x \cap \alpha'$ are the co-atoms of $ABS(B)$

3.The meaning of CPPs:

How it is possible that CPPs keep their general meaning constant across categories. I propose to explain this meaning constancy of CPPs across categories by relating their denotations to atomicity of corresponding denotational algebras. Thus, in the simplest case an expression with a CPP denotes an atom in the algebra whose type is determined by the category of the argument of the particle. Other particles denote Boolean combinations of atoms and, possibly, of "variables" of appropriate category. For instance expressions denoting co-atoms, that is Boolean complements of atoms, can also be considered as having a general, category independent meaning given that Boolean complements have such a meaning as well. Similarly a function of the form $f_c(x_c) = x_c \vee_c at_c$, can be considered as having a general meaning independent of category c because in its definition category independent operations are used.

Let us consider first the classical CPPs *only, also* and *even.* We observe that all these particles are semantically modifiers denoting restrictive functions. This means in particular that the sentences with a particle entail the corresponding "particle-less" sentence. Their meaning constancy is due to the fact that their denotations are linked to atomicity. The case of *only* is relatively easy. We can explain its meaning constancy across categories by saying that *only* always denotes atoms of the denotational algebras of modifiers (Zuber 2001). Which exact atom and in which algebra depends on the category and value of the argument of *only.* Thus *only* in *only NP* denotes an atom in $D_{NP/NP}$, *only* in *only yesterday* denotes an atom in $D_{VP/VP}$, *only* in *only five* denotes an atom in the denotational algebra of modifiers of numerals (or determiners), etc.

This proposal concerning the relationship between *only* and atomicity can be justified more easily for some categories than for others. One can give an "almost formal" proof that *only NP* denotes an atom of D_{NP} using the fact that there is an isomorphism between the algebra D_{INT} of intersective determiners and the algebra D_{NP} (Zuber 2001).

Let us see now some other particles. There are some arguments (Zuber 2004) showing that *also* is the Boolean complement of *only*:

$$ALSO(X) = ONLY'(X).$$

Indeed *not only Leo* cross-categorially entails *also Leo* and *also Leo* cross-categorially entails *not only Leo.*

CPP *even* can be analysed as denoting an atomic function of the algebra of restrictive non-absolute modifiers. As indicated above, such functions are determined by two indices: an element of the denotational algebra of arguments of *even* and an atom included in this element. When the arguments are NPs atoms of the corresponding denotational algebras are singletons containing a property as a unique element. We obtain this property by taking the property corresponding to the VP of the sentence in which the subject NP is modified by *even* and intersecting it with the property pragmatically incompatible with it. There are two arguments for such a move. First, a conjunction of two NPs modified by *even* is impossible: **even Leo and even Lea.* Second, quantified NPs with *even* exhibit quantifier constraint in the same way as exception NPs (which are related to atoms). Thus we do not have **most/*some students, except Leo; *most/*some students,even Leo* but we do have *every student except Leo; every student, even Leo.* Given this (9) can be analyzed as in (10):

(9) Even Leo danced

(10) $EVEN(L)\ DANCED = ONLY\ L\ IS\ D \cap Inc(D)$

(10) informally means that Leo is the only dancer who has a property incompatible with dancing.

Using the above description of classical CPPs we can define the meaning of other CPPs.

Thus the meaning of *et least* is given in (11) and the meaning of *at most* is given in (12):

(11) $AT - LEAST(X) = X\ OR\ NOT - ONLY(X)$
(12) $AT - MOST(X) = ONLY(X)\ OR\ NOT - EVEN(X)$

Notice that descriptions in (11) and (12) are category (type) independent. This means that the variable X above can be of any (major) category. For instance *at most Leo* "means" Only Leo or not even Leo.

4. Conditionals:

We can now apply the above description of CPPs to analyse conditionals modified by CPPs. Such an application is in principle independent of any particular theory of conditionals, even if a theory of conditionals in the framework of Boolean semantics would be more appropriate (cf. Zuber 2003).

Thus using the analysis above we get:

(13) P also if Q=P not only if Q
(14) P at least if Q=P if Q or P not only if Q
(15)*P at most if Q=P only if Q or P not even if Q

Notice that the description of conditional sentences with *at most* given in (15) indicates that they are uninformative hence probably their ungrammaticality. Furthermore, concerning *even* it follows from my proposal that *even if* conditionals do not entail their consequent (the consequent entailment thesis). This is because (16) does not entail that that Leo will dance (in the same way as (17) does not entail that everybody danced):

(16) Leo will dance if it rains and even if it snows
(17) Leo and even Lea will dance

5. Conclusions:

Using the Boolean semantics we analysed CPPs in an unified way which allows us to understand why such particles keep their meaning constant independently of the category of the argument to which they apply, even if they apply to CSs. This is possible because in the Boolean semantics one can naturally use category (type) independent notions such as Boolean operations and atoms. In this paper an additional attempt has been made to explain the surprise effect induced by some CPPs (*even, in particular*): it is proposed that the surprise effect is due to exceptionality of atomic elements. A full analysis of this problems necessitates additional tools since they seem to involve intensionality (Zuber 2006)

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