Structured discourse reference to propositions: Entailment particles and modal subordination in dynamic type logic

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1 The phenomenon

The empirical goal of this paper[1] is to provide a representation for the discourse in (1) below that assigns it the intuitively correct truth-conditions and that explicitly captures the anaphoric connections established in it.

(1) a. [A] man cannot live without joy.
b. Therefore, when he is deprived of true spiritual joys, it is necessary that he become addicted to carnal pleasures.
(Thomas Aquinas[2])

We are interested in the following features of this discourse. First, we want to capture the meaning of the entailment particle *therefore,* which relates the *content* of the premise (1a) and the *content* of the conclusion (1b) and requires the latter to be *entailed* by the former. I take the content of a sentence to be *truth-conditional* in nature, i.e., to be the set of possible worlds in which the sentence is true, and entailment to be *content inclusion,* i.e., (1a) entails (1b) iff for any world w, if (1a) is true in w, so is (1b).[3]

Second, we are interested in the *meanings* of (1a) and (1b). I take meaning to be *context-change potential,* i.e., to encode both *content* (truth-conditions) and *anaphoric potential.* Thus, on the one hand, we are interested in the *contents* of (1a) and (1b). They are both modal quantifications: (1a) involves a circumstantial modal base (to use

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[2] Attributed to Thomas Aquinas, `http://en.wikiquote.org/wiki/Thomas_Aquinas#Attributed`.

[3] I am grateful to a LoLa 9 reviewer for pointing out that modeling the entailment relation expressed by *therefore* as a truth-conditional relation, i.e., as requiring inclusion between two sets of possible worlds, cannot account for the fact that the discourse π *is an irrational number, therefore Fermat's last theorem is true* is not intuitively acceptable as a valid entailment and it cannot be accepted as a mathematical proof despite the fact that both sentences are necessary truths (i.e., they are true in every possible world). I think that at least some of the available accounts of hyper-intensional phenomena are compatible with my proposal, so I do not see this as an insurmountable problem.

the terminology introduced in Kratzer 1981) and asserts that, in view of the circumstances, i.e., given that God created men in a particular way, as long as a man is alive, he must find some thing or other pleasurable; (1b) involves the same modal base and elaborates on the preceding modal quantification: in view of the circumstances, if a man is alive and has no spiritual pleasure, he must have a carnal pleasure. Note that we need to make the contents of (1a) and (1b) accessible in discourse so that the entailment particle *therefore* can relate them.

On the other hand, we are interested in the *anaphoric potential* of (1a) and (1b), i.e., in the anaphoric connections between them. These connections are explicitly represented in discourse (2) below, which is intuitively equivalent to (1) albeit more awkwardly phrased. Indefinites introduce a discourse referent (dref) u_1, u_2 etc., which is represented by superscripting the dref, while pronouns are anaphoric to a dref, which is represented by a subscript.

(2) a. If a^{u_1} man is alive, he$_{u_1}$ must find something^{u_2} pleasurable/he$_{u_1}$ must have a^{u_2} pleasure.

b. Therefore, if he$_{u_1}$ doesn't have any^{u_3} spiritual pleasure, he$_{u_1}$ must have a^{u_4} carnal pleasure.

Note in particular that the indefinite *a man* in the antecedent of the conditional in (2a) introduces the dref u_1, which is anaphorically retrieved by the pronoun *he* in the antecedent of the conditional in (2b). This is an instance of **modal subordination** (Roberts 1989), i.e., an instance of simultaneous modal and invididual-level anaphora (see Frank 1996; Geurts 1999; Stone 1999): the conditional in (2b) covertly 'duplicates' the antecedent of the conditional in (2a), i.e., it asserts that, if *a man is alive* and doesn't have any spiritual pleasure, he must have a carnal one.

I will henceforth analyze the simpler and more transparent discourse in (2) instead of the naturally occurring discourse in (1). The challenge posed by (2) is that, when we *compositionally* assign meanings to (i) the modalized conditional in (2a), i.e., the premise, (ii) the modalized conditional in (2b), i.e., the conclusion; (iii) the entailment particle *therefore,* which relates the premise and the conclusion, we have to capture both the *intuitively correct truth-conditions* of the whole discourse and the modal and individual-level *anaphoric connections* between the two sentences of the discourse and within each one of them.

2 The basic proposal: Intensional Plural CDRT

To analyze discourse (1/2), I will introduce a new dynamic system couched in many-sorted type logic which extends Compositional DRT (CDRT) (see Muskens 1996) in two ways. In the spirit of the Dynamic Plural Logic of Van den Berg (1996), I model information states I, J etc. as *sets of variable assignments* i, j etc., and let sentences denote relations between such *plural* info states. In the spirit of Stone (1999), I analyze modal anaphora by means of dref's for *static* modal objects.[4] I will call the resulting system Intensional Plural CDRT (IP-CDRT). IP-CDRT takes the research program in Muskens (1996), i.e., the unification

[4] This is in contrast to Geurts (1999) and Frank (1996), among others, who use dref's for *contexts* (i.e., for info states) to analyze modal anaphora and, therefore: (i) complicate the architecture of the system, e.g., info states are not necessarily well-founded, and (ii) fail to capture the parallel between anaphora and quantification in the individual and the modal domain — see Stone (1999) and Schlenker (2005) among others for more discussion of this parallel. For a detailed comparison with the previous literature, see Brasoveanu (2006).

of Montague semantics and DRT, one step further: IP-CDRT unifies — in dynamic type logic — the static Lewis (1973)/Kratzer (1981) analysis of modal quantification and Van den Berg's dynamic plural logic.

We work with a Dynamic Ty3 logic. That is, following Muskens (1996), we extend Ty2 (Gallin 1975) — which has three basic types: t (truth-values), e (individuals; variables: x, x' etc.) and $\mathbf{w}$ (possible worlds; variables: w, w' etc.) — with a basic type s whose elements are meant to model variable assignments (variables of type s: i, j etc.). A suitable set of axioms ensures that i, j etc. behave like variable assignments in the relevant respects.[5] A dref for individuals u is a function of type se from 'assignments' i_s to individuals x_e; intuitively, the individual $u_{se}i_s$ is the individual that i assigns to the dref u. A dref for possible worlds p is a function of type $s\mathbf{w}$ from 'assignments' i_s to possible worlds $w_{\mathbf{w}}$; intuitively, the world $p_{s\mathbf{w}}i_s$ is the world that i assigns to the dref p.

Dynamic info states are sets of 'variable assignments', i.e., terms I, J etc. of type st. A sentence is interpreted as a DRS, i.e., as a relation of type $(st)((st)t)$ between an input and an output info state. An individual dref u stores a set of individuals with respect to an info state I, abbreviated $uI := \{u_{se}i_s : i_s \in I_{st}\}$. A dref p stores a set of worlds, i.e., a *proposition*, with respect to an info state I, abbreviated $pI := \{p_{s\mathbf{w}}i_s : i_s \in I_{st}\}$. *Propositional* dref's have two uses: (i) they store *contents*, e.g., the content of the premise (2a); (ii) they store *possible scenarios* (in the sense of Stone 1999), e.g., the set of worlds introduced by the conditional antecedent in (2a).

We use *plural info states* to store sets of individuals and propositions instead of simply using dref's for sets of individuals or possible worlds (their types would be $s(et)$ and $s(\mathbf{w}t)$) because we need to store in our discourse context (i.e., in our information states) both the *values* assigned to various dref's and the *structure* associated with those values. To see this, consider the example of plural anaphora in (3) below and the example of modal subordination in (4).

(3) a. Every^{u} man saw a$^{u'}$ woman.
    b. They$_{u}$ greeted them$_{u'}$.

(4) a. A^{u} wolf might^{p} enter the cabin.
    b. It$_{u}$ would$_{p}$ attack John.

In both cases, we do not simply have anaphora to sets, but anaphora to *structured* sets: if man m_1 saw woman n_1 and m_2 saw n_2, (3b) is interpreted as asserting that m_1 greeted n_1, not n_2, and that m_2 greeted n_2, not n_1; the structure of the greeting is the same as the structure of the seeing. Similarly, (4b) is interpreted as asserting that, if a wolf entered the cabin, it would attack John, i.e., if a black wolf x_1 enters the cabin in world w_1 and a white wolf x_2 enters the cabin in world w_2, then x_1 attacks John in w_1, not in w_2, and x_2 attacks John in w_2, not in w_1.

A plural info state I stores the *quantificational structure* associated with sets of individuals and possible worlds: (3a) requires each variable assignment $i \in I$ to be such that the man ui saw the woman $u'i$; (3b) elaborates on this structured dependency by requiring that, for each $i \in I$, the man ui greeted the woman $u'i$. Similarly, (4a) outputs an info state I such that, for each $i \in I$, the wolf ui enters the cabin in the world pi; (4b)

[5] Notational conventions: (i) subscripts on terms represent their types, e.g., x_e, $w_{\mathbf{w}}$, i_s; (ii) lexical relations are subscripted with their world variable, e.g., $\text{see}_w(x, y)$ is intuitively interpreted as 'x saw y in world w'.

elaborates on this structured dependency: for each assignment $i \in I$, it requires the wolf ui to attack John in world pi.

Moreover, we need plural info states to capture structured anaphora between the premise(s) and the conclusion of 'entailment' discourses like (1/2) above or (5) and (6) below.

(5) a. Every^u man saw a$^{u'}$ woman.
b. Therefore, they$_u$ noticed them$_{u'}$.

(6) a. A^u wolf might^p enter the cabin.
b. It$_u$ would$_p$ see John$^{u'}$.
c. Therefore, it$_u$ would$_p$ notice him$_{u'}$.

Let's return now to discourse (2), which is analyzed as shown in (7) below.

(7) **CONTENT**^{p_1}**:**
if^{p_2}(a^{u_1} man$_{p_2}$ is alive$_{p_2}$);
must$^{p_3}{}_{p_1,\mu,\omega}(p_2, p_3)$; he$_{u_1}$ has$_{p_3}$ a^{u_2} pleasure$_{p_3}$.
THEREFORE$^{p_4}{}_{p^*,\mu^*,\omega^*}(p_1, p_4)$:
if$(p_5 \Subset p_2;$ **not**(he$_{u_1}$ has$_{p_5}$ a^{u_3} spiritual pleasure$_{p_5}$));
must$^{p_6}{}_{p_4,\mu,\omega}(p_5, p_6)$; he$_{u_1}$ has$_{p_6}$ a^{u_4} carnal pleasure$_{p_6}$.

The representation in (7) is basically a network of structured anaphoric connections. Consider the conditional in (2a) first. The morpheme *if* introduces a dref p_2 that stores the content of the antecedent — we need this distinct dref because the antecedent in (2b) is anaphoric to it (due to modal subordination). The indefinite *a man* introduces an individual dref u_1, which is later retrieved: (i) by the pronoun *he* in the consequent of (2a), i.e., by 'donkey' anaphora, and (ii) by the pronoun *he* in the antecedent of (2b), i.e., by modal subordination.

The modal verb *must* in the consequent of (2a) contributes a tripartite quantificational structure and it relates three propositional dref's. The dref p_1 stores the content of the whole modalized conditional. The dref p_2, which was introduced by the antecedent and which is anaphorically retrieved by *must,* provides the restrictor of the modal quantification. Finally, p_3 is the nuclear scope of the modal quantification; it is introduced by the modal *must,* which constrains it to contain the set of *ideal* worlds among the p_2-worlds — ideal relative to the p_1-worlds, a *circumstantial* modal base (MB) μ and an *empty* ordering source (OS) ω. Finally, we test that the set of ideal worlds stored in p_3 satisfies the remainder of the consequent.

Consider now the entailment particle *therefore.* I take it to relate *contents* and not meanings. This is motivated by the entailment discourses in (5) and (6) above: in both cases, the contents (i.e., truth-conditions) of the premise(s) and the conclusion stand in an inclusion relation, but not their meanings (i.e., context change potentials). Further support is provided by the fact that the felicity of *therefore*-discourses is *context-dependent* — which is expected if *therefore* relates contents because contents are determined in a context-sensitive way. Consider, for example, the discourse in (8) below: entailment obtains if (8) is uttered on a Thursday in a discussion about John, but not otherwise.

(8) a. He$_{John}$ came back three days ago$_{Thursday}$.
b. Therefore, John came back on a Monday.

Moreover, I propose that *therefore* in (2b) should be analyzed as a modal relation, in particular, as expressing *logical consequence;* thus, I analyze discourse (1/2) as a modal quantification that relates two embedded modal quantifications, the second of which is modally subordinated to the first. Just as the modal *must, therefore* contributes a necessity modal relation and introduces a tripartite quantificational structure: the restrictor is p_1 (the content of the premise) and the nuclear scope is the newly introduced dref p_4, which stores the set of ideal p_1-worlds — ideal relative to the dref p^* (the designated dref for the actual world w^*), an *empty* MB μ^* and an *empty* OS ω^* (empty because *therefore* is interpreted as logical consequence). Since μ^* and ω^* are empty, the dref p_4 is identical to p_1.

Analyzing *therefore* as an instance of modal quantification makes at least two welcome predictions. First, it predicts that we can interpret it relative to different MB's and OS's — and this prediction is borne out.[6] Second, it captures the intuitive equivalence between the *therefore*-discourse *A man saw a woman, therefore he noticed her* and the modalized conditional *If a man saw a woman, he (obviously/necessarily) noticed her* (they are equivalent provided we add the premise *A man saw a woman* to the conditional).

The conditional in (2b) is interpreted like the conditional in (2a), with the additional twist that its antecedent is anaphoric to the antecedent of the conditional in (2a), i.e., to the dref p_2. The dref p_5 is a *structured* subset of p_2, symbolized as $p_5 \Subset p_2$. We need *structured inclusion* because we want p_5 to preserve the structure associated with the p_2-worlds, i.e., to preserve the association between p_2-worlds and the u_1-men in them. The modal verb *must* in (2b) is anaphoric to p_5, it introduces the set of worlds p_6 containing all the p_5-worlds that are ideal relative to the p_4-worlds, μ and ω (the same as the MB and OS in the premise (2a)) and it checks that in each ideal p_6-world, all its associated u_1-men have a carnal pleasure.

Over and above discourse (1/2), IP-CDRT can scale up to account for a wide range of examples, including the modal subordination example in (9) below from Roberts (1996).

(9) a. You should buy a lottery ticket and put it in a safe place.
  b. You're a person with good luck.
  c. It might be worth millions.

Note that the *might* modal quantification in (9c) is restricted by the content of the first conjunct below the modal *should* in (9a), i.e., it is interpreted as asserting that, given that you're a generally lucky person, *if you buy a lottery ticket*, it might be worth millions. Crucially, (9c) is not restricted by the content of both conjuncts in (9a) or by the set of deontically ideal worlds contributed by *should.*

Roberts (1996) proposes to analyze (9c) by *accommodating a suitable domain restriction* for the quantification contributed by *might.* The accommodation procedure, however, is left largely unspecified and unrestricted; moreover, it is far from clear that accommodation is right way to go when the relevant domain restriction is in fact provided

[6] *Therefore* expresses causal consequence in (i) below and a form of practical inference in (ii).

(i) Reviewers are usually people who would have been poets, historians, biographers, etc., if they could; they have tried their talents at one or the other, and have failed; therefore they turn critics. (Samuel Taylor Coleridge, *Lectures on Shakespeare and Milton.*)

(ii) We cannot put the face of a person on a stamp unless said person is deceased. My suggestion, therefore, is that you drop dead.
(Attributed to J. Edward Day; letter, never mailed, to a petitioner who wanted himself portrayed on a postage stamp.)

by the preceding discourse. In contrast, IP-CDRT provides the right kind of framework for an analysis of (9c) in terms of *content anaphora.* An anaphoric analysis of (9c) is desirable because it is more restricted than an accommodation account and because we can capture the connection between (9c) and the preceding discourse, i.e., (9a), in a simple and formally explicit way.

3 The outline of the formal IP-CDRT analysis

In a Fregean/Montagovian framework, the compositional aspect of interpretation is largely determined by the types for the extensions of the 'saturated' expressions, i.e., names and sentences, plus the type that allows us to build intensions out of these extensions. Let's abbreviate them as **e**, **t** and **s**, respectively. In IP-CDRT, we assign the following dynamic types to the 'meta-types' **e**, **t** and **s**: a sentence is interpreted as a DRS, i.e., as a relation between info states, hence $\mathbf{t} := (st)((st)t)$; a name is interpreted as an individual dref, hence $\mathbf{e} := se$; finally, $\mathbf{s} := s\mathbf{w}$, i.e., we use the type of propositional dref's to build intensions.

To interpret a noun like *man,* we define a dynamic relation $\text{man}_p\{u\}$ based on the static one $\text{man}_w(x)$, i.e.,

$$\text{man}_p\{u\} := \lambda I_{st}.I \neq \emptyset \wedge \forall i_s \in I(\text{man}_{pi}(ui)).$$

These dynamic relations are the counterpart of DRT's conditions. A sentence (type **t**) is represented as a linearized DRS (a.k.a. linearized box), i.e.,

$$[\textbf{new drefs,} \text{ e.g., } u, p \mid \textbf{conditions,} \text{ e.g., } \text{man}_p\{u\}].$$

A linearized DRS is the abbreviation of a term of the form

$$\lambda I_{st}\lambda J_{st}.I[\textbf{new drefs}]J \wedge \textbf{conditions}(J),$$

which states that the output info state J differs from the input info state I at most with respect to the **new drefs**[7] and each **condition** is satisfied in the output state J. A DRS that does not introduce any new dref's is represented as

$$[\textbf{conditions}] := \lambda I_{st}\lambda J_{st}.I = J \wedge \textbf{conditions}(J).$$

The noun *man* is translated as a term of type $\mathbf{e}(\mathbf{st})$:

$$man \rightsquigarrow \lambda v_{\mathbf{e}}\lambda q_{\mathbf{s}}.[\text{man}_q\{v\}].$$

Determiners are relations-in-intension between a property $P'_{\mathbf{e}(\mathbf{st})}$ (the restrictor) and another property $P_{\mathbf{e}(\mathbf{st})}$ (the nuclear scope). Indefinite determiners, e.g., a^u, introduce an individual dref u and check that the dref satisfies the restrictor and the nuclear scope:

$$a^u \rightsquigarrow \lambda P'_{\mathbf{e}(\mathbf{st})}\lambda P_{\mathbf{e}(\mathbf{st})}\lambda q_{\mathbf{s}}.[u]; P'(u)(q); P(u)(q).$$

[7] The definition of $I[\varrho]J$ (for some dref ϱ) is

$$\forall i_s \in I(\exists j_s \in J(i[\varrho]j)) \wedge \forall j_s \in J(\exists i_s \in I(i[\varrho]j));$$

for its empirical and theoretical justification, see Brasoveanu (2006).

The semi-colon ';' is dynamic conjunction, interpreted as relation composition:

$$D; D' := \lambda I_{st}\lambda J_{st}.\exists H_{st}(DIH \wedge DHJ).$$

A pronoun he_u is anaphoric to an individual dref u and is translated as the Montagovian type-lift of the dref u:

$$he_u \rightsquigarrow \lambda P_{\mathbf{e(st)}}.P(u).$$

Given fairly standard assumptions about Logical Forms (LF's) and type-driven translation, a simple sentence like A^{u_1} *man is alive* is compositionally translated as

$$\lambda q_{\mathbf{s}}.[u_1|\mathrm{man}_q\{u_1\}, \mathrm{alive}_q\{u_1\}].$$

I assume that the LF of such a sentence contains an indicative mood morpheme $\mathbf{ind}_{p^*}$ whose meaning is $\lambda \mathbb{P}_{\mathbf{st}}.\mathbb{P}(p^*)$, i.e., it takes the dynamic proposition $\mathbb{P}_{\mathbf{st}}$ denoted by the remainder of the sentence and applies it to the designated dref for the actual world p^*.

To interpret the conditional in (2a) above, we need to: (i) extract the content of the antecedent of the conditional and store it in a propositional dref p_2 and (ii) define a dynamic notion of *structured* subset of a set of worlds. Let's start with (ii). We need a notion of structured inclusion because: (a) the modal *must* and the 'donkey' pronoun *he* in the consequent of (2a) are simultaneously anaphoric to the p_2-worlds and the u_1-men and we need to preserve the structured dependencies between them; (b) the modally subordinated antecedent of the conditional in (2b) is also anaphoric to p_2 and u_1 in a structured way. In the spirit of Van den Berg (1996), I will assume that there is a dummy world # (of type **w**) relative to which all lexical relations are false and I will use this world to define the **structured inclusion condition**

$$p \Subset p' := \lambda I_{st}.I \neq \emptyset \wedge \forall i_s \in I(pi = p'i \vee pi = \#).$$

The dummy world # is used to signal that an 'assignment' i such that $pi = \#$ is irrelevant for the evaluation of conditions, so we need to slightly modify the definition of conditions:

$$\mathrm{man}_p\{u\} := \lambda I_{st}.I_{p\neq\#} \neq \emptyset \wedge \forall i_s \in I_{p\neq\#}(\mathrm{man}_{pi}(ui)),$$

where $I_{p\neq\#} := \{i_s \in I\colon pi \neq \#\}$.

To extract the content of the antecedent of the conditional, we define two operators over a propositional dref p and a DRS D: a maximization operator $\mathbf{max}^p(D)$ and a distributivity operator $\mathbf{dist}_p(D)$.[8] These operators enable us to 'dynamize' λ-abstraction over possible worlds, i.e., to extract and store contents: the $\mathbf{dist}_p(D)$ update checks one world at a time that the set of worlds stored in p satisfies the DRS D and the $\mathbf{max}^p(D)$ update collects in p all the worlds that satisfy D. Thus, we translate *if* as:

$$if^{p_2} \rightsquigarrow \lambda \mathbb{P}_{\mathbf{st}}.\mathbf{max}^{p_2}(\mathbf{dist}_{p_2}(\mathbb{P}(p_2))).$$

[8] The definitions in (i) and (ii) below follow the basic ideas, but not the exact definitions, of the corresponding operators over individual dref's in Van den Berg (1996). The definition of $\mathbf{dist}_p(D)$ incorporates an amendment of Van den Berg's definition proposed in Nouwen (2003).

(i) $\mathbf{max}^p(D) := \lambda I_{st}\lambda J_{st}.([p]; D)IJ \wedge \forall K_{st}(([p]; D)IK \rightarrow pK \subseteq pJ)$;

(ii) $\mathbf{dist}_p(D) := \lambda I_{st}\lambda J_{st}.pI = pJ \wedge I_{p=\#} = J_{p=\#} \wedge \forall w \in pI_{p\neq\#}(DI_{p=w}J_{p=w})$,
where $I_{p=w} := \{i_s \in I\colon pi = w\}$.

We need one last thing to translate the antecedent in (2a). The 'donkey' indefinite *a man* receives a *strong* reading, i.e., the conditional in (2a) is interpreted as asserting that *every* (and not only *some*) man that is alive must have a pleasure. However, the meaning for indefinite determiners given above incorrectly assigns a *weak* reading to the indefinite. I will analyze indefinite determiners as ambiguous between a weak and a strong meaning and I define the strong meaning in terms of **max**:

$$a^{\mathbf{str}:u} \rightsquigarrow \lambda P'_{\mathbf{e(st)}} \lambda P_{\mathbf{e(st)}} \lambda q_{\mathbf{s}}.\mathbf{max}^{u}(P'(u)(q); P(u)(q)).$$[9]

So, the antecedent of the conditional in (2a) is translated as:

$$if^{p_2}\ a^{\mathbf{str}:u_1}\ man\ is\ alive \rightsquigarrow \mathbf{max}^{p_2}(\mathbf{dist}_{p_2}(\mathbf{max}^{u_1}([\mathrm{man}_{p_2}\{u_1\}, \mathrm{alive}_{p_2}\{u_1\}]))).$$

The modal verb *must* is interpreted in terms of a modal condition $\mathbf{nec}_{p,\mu,\omega}(p', p'')$. The condition is relativized to: (i) a propositional dref p storing the content of the entire modal quantification, (ii) an MB dref μ and (iii) an OS dref ω. Both μ and ω are dref's for sets of worlds, i.e., they are of type $s(\mathbf{w}t)$, a significant simplification compared to the type of static MB's and OS's in Kratzer (1981), i.e., $\mathbf{w}((\mathbf{w}t)t)$.[10,11] So, *must* is translated as follows:

$$\begin{aligned} must^{p_3 \Subset p_2}{}_{p_1,\mu,\omega} \rightsquigarrow \lambda \mathbb{P}_{\mathbf{st}}.&[\mu, \omega | \mathbf{circumstantial}_{p^*}\{p_1, \mu\}, \mathbf{empty}\{p_1, \omega\}];\\ &[p_3 | \mathbf{nec}_{p_1,\mu,\omega}(p_2, p_3)]; \mathbf{dist}_{p_3}(\mathbb{P}(p_3)). \end{aligned}$$

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[9] Brasoveanu (2006) provides extensive motivation for this analysis of the weak/strong 'donkey' ambiguity.

[10] We can simplify these types in IP-CDRT because we have plural info states: every world $w \in pI$ is associated with a sub-state $I_{p=w}$ and we can use this sub-state to associate a set of propositions with the world w, namely the set of propositions $\{\mu i\colon i_s \in I_{p=w}\}$, where each μi is of type $\mathbf{w}t$. I take the dummy value for MB and OS dref's to be the singleton set whose member is the dummy world, i.e., $\{\#\}$.

[11] $\mathbf{nec}_{p,\mu,\omega}(p', p'') := \lambda I_{st}.I_{p\neq\#} \neq \emptyset \wedge$
$\forall w \in pI_{p\neq\#}(\mathbf{NEC}_{\mu I_{p=w,\mu\neq\{\#\}}, \omega I_{p=w,\omega\neq\{\#\}}}(p' I_{p=w,p'\neq\#}, p'' I_{p=w,p''\neq\#})) \wedge$
$(p'' \Subset p')I \wedge \forall w \in pI_{p\neq\#} \forall i \in I_{p=w}(p'i \in p'' I_{p=w,p''\neq\#} \rightarrow p'i = p''i)$.

NEC is the static modal relation, basically defined as in Lewis (1973) and Kratzer (1981). The dref's μ and ω associate with each p-world two sets of propositions M and O of type $(\mathbf{w}t)t$. The set of propositions O induces a strict partial order $<_O$ on the set of all possible worlds as shown in (i) below. I assume that all the strict partial orders of the form $<_O$ satisfy the Generalized Limit Assumption in (ii) — therefore, the **Ideal** function in (iii) is well-defined. This function extracts the subset of O-ideal worlds from the set of worlds W. Possibility modals are interpreted in the same way, we only need to replace **NEC** with **POS**; both are defined in (iv) below.

(i) $w <_O w'$ iff $\forall W \in O(w' \in W \rightarrow w \in W) \wedge \exists W \in O(w \in W \wedge w' \notin W)$

(ii) **Generalized Limit Assumption:** for any proposition $W_{\mathbf{w}t}$ and OS $O_{(\mathbf{w}t)t}$,

$$\forall w \in W \exists w' \in W((w' <_O w \vee w' = w) \wedge \neg\exists w'' \in W(w'' <_O w'))$$

(iii) For any proposition $W_{\mathbf{w}t}$ and OS $O_{(\mathbf{w}t)t}$:

$$\mathbf{Ideal}_O(W) := \{w \in W\colon \neg\exists w' \in W(w' <_O w)\}$$

(iv) $\mathbf{NEC}_{M,O}(W_1, W_2) := W_2 = \mathbf{Ideal}_O((\cap M) \cap W_1)$;
$\mathbf{POS}_{M,O}(W_1, W_2) := W_2 \neq \emptyset \wedge W_2 \subseteq \mathbf{Ideal}_O((\cap M) \cap W_1)$.

We introduce the modal base μ and the ordering source ω and relate them to the dref p_1 (which stores the content of the modal quantification) by the **circumstantial** and **empty** conditions.[12] The condition $\mathbf{circumstantial}_{p^*}\{p_1, \mu\}$ is *context-dependent,* i.e. it is relativized to the dref for the actual world p^*; we need this because the argument in (1/2) goes through only if we add another premise to the one explicitly stated, namely that pleasures are spiritual or carnal. That is, the condition $\mathbf{circumstantial}_{p^*}\{p_1, \mu\}$ is meant to constrain the modal quantification in the premise (2a) so that it is evaluated only with respect to worlds whose circumstances are identical to the actual world w^* in the relevant respects — in particular, the proposition

$$\{w_{\mathbf{w}} : \forall x_e(\text{pleasure}_w(x) \rightarrow \text{spiritual}_w(x) \vee \text{carnal}_w(x))\}$$

has to be true in these worlds just as it is in w^*.

Like *must,* the particle *therefore* introduces a necessity quantificational structure. Since *therefore* expresses logical consequence, both its MB μ^* and its OS ω^* are empty:

$$\begin{aligned} \textit{therefore}^{p_4 \Subset p_1}{}_{p^*,\mu^*,\omega^*} \rightsquigarrow \lambda \mathbb{P}_{\mathbf{st}}.&[\mu^*, \omega^* | \mathbf{empty}\{p^*, \mu^*\}, \mathbf{empty}\{p^*, \omega^*\}];\\ &[p_4 | \mathbf{nec}_{p^*,\mu^*,\omega^*}(p_1, p_4)]; \mathbf{dist}_{p_4}(\mathbb{P}(p_4)). \end{aligned}$$

The effect of the update is that the dref p_4 is identical to p_1 both in its value and in its structure, i.e., if J is the output state after processing the **nec** condition, we have that $p_1 j = p_4 j$ for any 'assignment' $j \in J$. Consequently, p_1 can be freely substituted for p_4. I assume that the anaphoric nature of the entailment particle *therefore,* which requires a propositional dref p_1 as the restrictor of its quantification, triggers the accommodation of a *covert* 'content-formation' morpheme $\textit{if}^{p_1}$ that takes scope over the premise (2a) and stores its content in p_1.

The conditional in (2b) is different from the one in (2a) in three important respects. First, given that (2b) elaborates on the modal quantification in (2a), the modal verb *must* in (2b) is anaphoric to the previously introduced MB μ (circumstantial) and OS ω (empty), so it is translated as

$$\textit{must}^{p_6 \Subset p_5}{}_{p_1,\mu,\omega} \rightsquigarrow \lambda \mathbb{P}_{\mathbf{st}}.[p_6 | \mathbf{nec}_{p_1,\mu,\omega}(p_5, p_6)]; \mathbf{dist}_{p_6}(\mathbb{P}(p_6)).$$

Second, the negation in the antecedent of (2b) is translated as

$$\textit{not} \rightsquigarrow \lambda \mathbb{P}_{\mathbf{st}} \lambda q_{\mathbf{s}}.[\sim\mathbb{P}(q)],$$

i.e., in terms of the dynamic negation $\sim D$.[13] Finally, the modally subordinated antecedent in (2b) is translated in terms of an update requiring the newly introduced dref p_5 to be a *maximal structured* subset of p_2, i.e.,

$$\textit{if}^{p_5 \Subset p_2} \rightsquigarrow \lambda \mathbb{P}_{\mathbf{st}}.\mathbf{max}^{p_5 \Subset p_2}(\mathbf{dist}_{p_5}(\mathbb{P}(p_5))).$$[14]

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[12] Definitions:

(i) $\mathbf{circumstantial}_p\{p', \mu\} := \lambda I_{st}.I_{p \neq \#, p' \neq \#} \neq \emptyset \wedge$
$\forall w \in pI_{p \neq \#}(\forall w' \in p'I_{p=w,p' \neq \#}(\mathbf{circumstantial}_w(w', \mu I_{p=w,p'=w'})).$

(ii) $\mathbf{empty}\{p, \omega\} := \lambda I_{st}.I_{p \neq \#} \neq \emptyset \wedge \forall i_s \in I(\omega i = \{\#\});$
$\mathbf{empty}\{p, \mu\} := \lambda I_{st}.I_{p \neq \#} \neq \emptyset \wedge \forall i_s \in I(\mu i = \{\#\}).$

[13] $\sim D := \lambda I_{st}.I \neq \emptyset \wedge \forall H_{st}(H \neq \emptyset \wedge H \subseteq I \rightarrow \neg \exists K_{st}(DHK))$; see Brasoveanu (2006) for detailed justification.

The IP-CDRT translation of the entire discourse (1/2) is provided in (10) below (for simplicity, I omit some distributivity operators and the modal conditions contributed by *therefore*) and, given the familiar dynamic definition of truth,[15] the discourse is assigned the intuitively correct truth-conditions.

(10) $\mathbf{max}^{p_1}(\mathbf{dist}_{p_1}(\mathbf{max}^{p_2}(\mathbf{dist}_{p_2}(\mathbf{max}^{u_1}([\mathrm{man}\{u_1\}, \mathrm{alive}\{u_1\}]))));$
$[\mu, \omega | \mathbf{circumstantial}_{p^*}\{p_1, \mu\}, \mathbf{empty}\{p_1, \omega\}]; [p_3 | \mathbf{nec}_{p_1,\mu,\omega}(p_2, p_3)];$
$[u_2 | \mathrm{pleasure}_{p_3}\{u_2\}, \mathrm{have}_{p_3}\{u_1, u_2\}]));$
$\mathbf{dist}_{p_1}(\mathbf{max}^{p_5 \Subset p_2}([\sim[u_3 | \mathrm{spiritual}_{p_5}\{u_3\}, \mathrm{pleasure}_{p_5}\{u_3\}, \mathrm{have}_{p_5}\{u_1, u_3\}]]);$
$[p_6 | \mathbf{nec}_{p_1,\mu,\omega}(p_5, p_6)]; [u_4 | \mathrm{carnal}_{p_6}\{u_4\}, \mathrm{pleasure}_{p_6}\{u_4\}, \mathrm{have}_{p_6}\{u_1, u_4\}]).$

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[14] $\mathbf{max}^{p \Subset p'}(D) := \lambda I_{st} \lambda J_{st}.\exists H([p | p \Subset p'] IH \wedge DHJ \wedge$
$\forall K_{st}([p | p \Subset p'] IK \wedge \exists L_{st}(DKL) \rightarrow K_{p \neq \#} \subseteq H_{p \neq \#}))$.

[15] **Truth:** A DRS D (type $\mathbf{t}$) is **true** with respect to an input info state I_{st} iff $\exists J_{st}(DIJ)$.