Be articulate!
A pragmatic solution to the projection problem

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0 Introduction

Heim (1983) suggested that the analysis of presupposition projection requires that the classical notion of **meanings as truth conditions** be replaced with a dynamic notion of **meanings as Context Change Potentials.**[1] But as several researchers later noted, the dynamic framework is insufficiently predictive: it allows one to state that, say, the dynamic effect of F *and* G is to *first* update a Context Set C with F and *then* with G (i.e., $C[F$ *and* $G] = C[F][G]$), but it fails to explain why there couldn't be a 'deviant' conjunction *and** which performed these operations in the opposite order (i.e., $C[F$ *and** $G] = C[G][F]$) (Soames 1989; Heim 1990, 1992). We provide a formal introduction to a competing framework, the **Transparency** theory (Schlenker 2006b), which addresses this problem. Unlike dynamic semantics, our analysis is *fully classical,* i.e., bivalent and static. And it *derives* the projective behavior of connectives from their bivalent meaning and their syntax. We concentrate on the formal properties of a simple version of the theory, and we prove that (i) full equivalence with Heim's results is guaranteed in the propositional case (Theorem 1), and that (ii) the equivalence can be extended to the quantificational case (for any generalized quantifiers), but only when certain conditions are met (Theorem 2).

1 The Transparency theory

The intuition we pursue is that the presupposition p of a clause $\underline{p}p'$ is simply a distinguished part of a bivalent meaning, one which is conceptualized as a 'pre-condition' of the entire meaning. We do not seek to explain how certain parts of the meaning of a constituent are conceptualized as being its 'pre-conditions'. This is another form of the old 'triggering problem' for presuppositions, i.e., the problem of determining how elementary clauses come to have presuppositions to begin with. Since we are interested in the projection problem rather than in the triggering problem, we simply stipulate in the syntax of the object language that a clause represented as $\underline{p}p'$ has the truth-conditional content of the conjunction p *and* p', but that p is conceptualized as being the pre-condition of the entire meaning. On the other hand, our goal is to give an explanatory account of presupposition projection. The crucial intuition is that a general pragmatic principle (presumably a Gricean maxim of manner, which we call *Be articulate!*) requires that, *if possible,* the special status of the pre-condition should be articulated, and thus that one should say p *and* $\underline{p}p'$ rather than just $\underline{p}p'$. To illustrate, the principle requires that, *if*

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possible, one should say *It is raining and John knows it* rather than just *John knows that it is raining.*

If we were to stop here, we would make the absurd prediction that *John knows that p* is never acceptable unless immediately preceded by *p and* ___. But there are independent pragmatic conditions that sometimes rule out the full conjunction. *It is precisely when these conditions are met that John knows that p is acceptable on its own.* In this paper we will only consider cases in which the full conjunction is ruled out *because the utterance of the first conjunct is certain to be dispensable no matter what the end of the sentence turns out to be* (see Schlenker 2006c for a sketch of further conditions, with several new predictions). This constraint is motivated by facts that have nothing to with presupposition projection:

(1)   a. Context: Everyone is aware that Pavarotti has cancer.
      i. *?Pavarotti is sick and he won't be able to sing next week.*
      ii. *Pavarotti won't be able to sing next week.*
   b. Context: Nothing is assumed about Pavarotti's health.
      i. # *Pavarotti has cancer and he is sick and he won't be able to sing next week.*
      ii. *Pavarotti has cancer and he won't be able to sing next week.*
   c. Context: Nothing is assumed about Pavarotti's health.
      i. # *If Pavarotti has cancer, he is sick and he won't be able to sing next week.*
      ii. *If Pavarotti has cancer, he won't be able to sing next week.*

The infelicitous examples are all cases in which one can determine as soon as one has heard *Pavarotti is sick and* that *no matter how the sentence will end*, these four words will have been uttered in vain because they could not possibly affect the truth-conditions of the sentence relative to the Context Set. Specifically, in a Context Set C in which it is assumed that Pavarotti has cancer, we can be sure that no matter what the second conjunct γ is, *Pavarotti is sick and* γ is equivalent in C to γ. We will say that given C these two sentences are **contextually equivalent** (i.e., $C \models (Pavarotti\ is\ sick\ and\ \gamma) \Leftrightarrow \gamma$). Similarly, in any Context Set in which it is assumed that cancer is a disease, *Pavarotti has cancer and he is sick and* γ is contextually equivalent to *Pavarotti has cancer and* γ; and by the same reasoning, *If Pavarotti has cancer, he is sick and* γ is contextually equivalent to *If Pavarotti has cancer,* γ. In all these cases, then, one can ascertain as soon as one has heard *he is sick and* that these words were uttered in vain. Any reasonable pragmatics should presumably rule this out, as suggested by (1) above.[2]

These observations lead us to the following definition:

(2)   Given a Context Set C, a predicative or propositional occurrence of d is **transparent** (and hence infelicitous) at the beginning of a sentence α *(d and* just in case for any expression γ of the same type as d and for any sentence completion β, $C \models \alpha(d\ and\ \gamma)\beta \Leftrightarrow \alpha\gamma\beta$.

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[2] Note, however, that we don't want to make the prohibition against redundant material too strong. For it is sometimes permissible to include a conjunct that turns out to be dispensable, but just in case one may only determine *later* in the sentence that the conjunct in question was eliminable. This scenario is illustrated in (i):

Our observations can now be summarized by noting that $\alpha(d \ and \ d')\dots$ is semantically deviant if d is transparent. Going back to the analysis of presupposition, it is clear that when d is transparent, a full conjunction *(d and $\underline{d}d'$)* will be systematically ruled out, which will leave $\underline{d}d'$ as the sole contender, and thus as the 'winner' in the competition process. Assuming for simplicity that Transparency is the *only* pragmatic principle that can rule out a full conjunction *(d and $\underline{d}d'$),* we are finally led to our formula for presupposition projection:

(3) **Principle of Transparency**
   Given a Context Set C, a predicative or propositional occurrence of $\underline{d}d'$ is acceptable at the beginning of a sentence $\alpha \underline{d}d'$

   if and only if the 'articulated' competitor $\alpha(d \ and \ \underline{d}d')$ is ruled out because d is transparent;

   if and only if for any expression γ of the same type as d and for any sentence completion β, $C \models \alpha(d \ and \ \gamma)\beta \Leftrightarrow \alpha\gamma\beta$.

We now show that the Principle of Transparency is sufficient to derive almost all of Heim's projection results (in ongoing research (Schlenker 2006c), we explore extensions of Transparency which make different predictions from Heim's and address some of the criticisms that were raised against her account).

2 Formal systems

2.1 Syntax

To make our analysis precise, we define a syntax in which the presuppositions of atomic clauses are underlined (the parts in bold do not belong to the object language but will be used in the meta-language):

(4) **Syntax**

   - Generalized Quantifiers: $Q ::= Q_i$;
   - Predicates: $P ::= P_i \mid \underline{P_i}P_k \mid (\mathbf{P}_i \ \textbf{and} \ \mathbf{P}_k)$;
   - Propositions: $p ::= p_i \mid \underline{p_i}p_k$;
   - **Individual variables: $\mathbf{d} ::= \mathbf{d}_i$;**
   - Formulas: $F ::= p \mid (not \ F) \mid (F \ and \ F) \mid (F \ or \ F) \mid (if \ F. \ F) \mid (Q_iP.P)$
     $\mid \mathbf{P(d)} \mid \forall\mathbf{dF} \mid \exists\mathbf{dF} \mid [\mathbf{F} \Rightarrow \mathbf{F}] \mid [\mathbf{F} \Leftrightarrow \mathbf{F}]$.

Terminology: We will say that p_i, $\underline{p_i}p_k$ are 'atomic propositions' and that P_i, $\underline{P_i}P_k$ are 'atomic predicates'.
   The following Lemma will be useful (the proof is omitted for brevity):

(i) a. John resides in France and he lives in Paris.

   b. If he is in Europe, John resides in France and he lives in Paris.

In both examples the contextual meaning of the sentence would be unaffected if we deleted the words *John resides in France and.* However this is something that can only be ascertained *after* one has heard the end of the sentence. Thus in (b) one needs to hear the entire sentence to determine that the first conjunct *John resides in France* was redundant (if the end of the sentence had been … *and he is happy,* the first conjunct would not have been redundant).

(5) **Syntactic Lemma**

    a. If α is the beginning of a constituent in a string F, then α is the beginning of a constituent in any well-formed string that contains α.

    b. If a formula F starts with $(s$, where s is a symbol different from a parenthesis, then the smallest initial string C of F which is a constituent is F itself.

2.2 Semantics

2.2.1 Framework and interpretation of lexical items

We define the semantics for a (possibly infinite) domain of possible worlds W, each of which has a domain of individuals D^w of a fixed finite size n. We write $[A \rightarrow B]$ to denote the set of functions with domain A and codomain B, and we use standard type-theoretic notation wherever useful (e.g., $\langle s, t \rangle$ is the type of propositions, i.e., of functions from possible worlds to truth values; and $\langle s, \langle e, t \rangle \rangle$ is the type of properties, i.e., of functions from possible worlds to characteristic functions of sets).

(6) **Interpretation of lexical items**
    We define a static interpretation function I. For all $i \geq 0$,

    a. Q_i is a generalized quantifier satisfying Permutation Invariance, Extension and Conservativity (Keenan 1996). Its value is entirely determined by a numerical function f_i in $[\mathbb{N} \times \mathbb{N} \rightarrow \{0, 1\}]$, which we call the 'tree of numbers' of Q_i (van Benthem 1986). Thus for all $w \in W$, $I_w(Q_i)$ is of type $\langle \langle e, t \rangle, \langle \langle e, t \rangle, t \rangle \rangle$, and for all A, B of type $\langle e, t \rangle$, $I_w(Q_i)(A)(B) = 1$ iff $f_i(|A \setminus B|, |A \cap B|) = 1$;

    b. $I_w(P_i) \in [W \rightarrow [D \rightarrow \{0, 1\}]]$ (i.e., it is of type $\langle s, \langle e, t \rangle \rangle$);

    c. $I_w(p_i) \in [W \rightarrow \{0, 1\}]$ (i.e., it is of type $\langle s, t \rangle$).

Notation: We write F^w instead of $I_w(F)$. When some elements are optionally present in the syntax, we write them between curly brackets, and write the corresponding part of the truth conditions inside curly brackets as well.

2.2.2 Dynamic semantics

Next, we define a dynamic semantics which is precisely that of Heim (1983), augmented by the analysis of disjunction offered in Beaver (2001) (Heim did not discuss disjunction).

(7) **Dynamic (Trivalent) Semantics**
    Let C be a subset of W.

- $C[p] = \{w \in C : p^w = 1\}$;
- $C[\underline{pp'}] = \#$ iff for some $w \in C$, $p^w = 0$; otherwise, $C[\underline{pp'}] = \{w \in C : p'^w = 1\}$;
- $C[(not\ F)] = \#$ iff $C[F] = \#$; otherwise, $C[(not\ F)] = C \setminus C[F]$;
- $C[(F\ and\ G)] = \#$ iff $C[F] = \#$ or $(C[F] \neq \# \wedge C[F][G] = \#)$; otherwise, $C[(F\ and\ G)] = C[F][G]$;
- $C[(F\ or\ G)] = \#$ iff $C[F] = \#$ or $(C[F] \neq \# \wedge C[not\ F][G] = \#)$; otherwise, $C[(F\ or\ G)] = C[F] \cup C[not\ F][G]$;
- $C[(if\ F.\ G)] = \#$ iff $C[F] = \#$ or $(C[F] \neq \# \wedge C[F][G] = \#)$; otherwise, $C[(if\ F.\ G)] = C \setminus C[F][not\ G]$;

- $C[(Q_i\{\underline{P}\}P'.\{\underline{R}\}R') = \#$ iff {for some $w \in C$, for some $d \in D$, $\underline{P}^w(d) = 0$} or {for some $w \in C$, for some $d \in D$, $\{\underline{P}^w(d) = 1$ and} $P'^w(d) = 1$ and $\underline{R}^w(d) = 0$}. Otherwise, $C[(Q_i\{\underline{P}\}P'.\{\underline{R}\}R')] = \{w \in C: f_i(a^w, b^w) = 1\}$ with $a^w = \{d \in D : P'^w(d) = 1 \wedge R'^w(d) = 0\}, b^w = \{d \in D: P'^w(d) = 1 \wedge R'^w(d) = 1\}$.

2.2.3 Static semantics

Sincd our goal is to show that the results of Heim's dynamic semantics can be obtained in a fully classical logic, we should specify a classical interpretation for the language defined in Section 2.1.

(8) **Static (Bivalent) Semantics**

- $w \models p$ iff $p^w = 1$;
- $w \models \underline{p}p'$ iff $p^w = p'^w = 1$;
- $w \models (\textit{not } F)$ iff $w \not\models F$;
- $w \models (F \textit{ and } G)$ iff $w \models F$ and $w \models G$;
- $w \models (F \textit{ or } G)$ iff $w \models F$ or $w \models G$;
- $w \models (\textit{if } F.\ G)$ iff $w \not\models F$ or $w \models G$;
- $w \models (Q_i\{\underline{P}\}P'.\{\underline{Q}\}Q'$ iff $f_i(a^w, b^w) = 1$ with $a^w = \{d \in D: \{\underline{P}^w(d) = 1$ and} $P'^w(d) = 1$ and $(\{\underline{R}^w(d) = 0$ or} $R'^w(d) = 0)\}, b^w = \{d \in D: \{\underline{P}^w(d) = 1$ and} $P'^w(d) = 1$ and $\{\underline{R}^w(d) = 1$ and} $R'^w(d) = 1\}$.

3 Propositional case

We now prove that in the propositional case Transparency Theory is equivalent to Heim's system. We assume that the language is sufficiently expressive to include tautologies and contradictions.

Theorem 1

Consider the propositional fragment of the language defined above. For any formula F and for any $C \subseteq W$:

(i) $\mathrm{Transp}(C, F)$ iff $C[F] \neq \#$.

(ii) If $C[F] \neq \#$, $C[F] = \{w \in C: w \models F\}$.

We start with a useful lemma (the proofs are omitted for brevity):

(9) **Transparency Lemma**

a. If for some formula G and some sentence completion δ, $\mathrm{Transp}(C, (G\delta)$, then $\mathrm{Transp}(C, G)$.

b. If for some formula G and some sentence completion δ, $\mathrm{Transp}(C, (\textit{if } G\ .\ \delta)$, then $\mathrm{Transp}(C, G)$.

We can now proceed to the proof of Theorem 1 (by induction on the construction of formulas).

a. $F = p$

(i) $C[F] \neq \#$ and $\mathrm{Transp}(C, F)$.

(ii) It is also clear that $C[F] = \{w \in C : p^w = 1\} = \{w \in C : w \models F\}$.

b. $F = \underline{pp'}$.

(i) If $\mathrm{Transp}(C, F)$, for any formula γ and for any sentence completion β,

$$C \models (p \ and \ \gamma)\beta \Leftrightarrow \gamma\beta,$$

hence in particular $C \models (p \ and \ \delta) \Leftrightarrow \delta$ for some tautology δ, and thus $C \models p$. Therefore $C[F] \neq \#$.

Conversely, if $C[F] \neq \#$, $C \models p$ and thus for any clause γ, $C \models (p \ and \ \gamma) \Leftrightarrow \gamma$. It follows that for any clause γ and for any sentence completion β, $C \models (p \ and \ \gamma)\beta \Leftrightarrow \gamma\beta$.[3]But this shows that $\mathrm{Transp}(C, F)$.

(ii) If $C[F] \neq \#$, $C \models p$ and $C[F] = \{w \in C : p'^w = 1\} = \{w \in C : p'^w = p^w = 1\} = \{w \in C : w \models \underline{pp'}\}$.

c. $F = (not \ G)$.

(i) Suppose that $\mathrm{Transp}(C, F)$ and suppose, for contradiction, that $C[F] = \#$. Then $C[G] = \#$ and by the Induction Hypothesis not $\mathrm{Transp}(C, G)$, i.e., for some initial string $\alpha\underline{dd'}$ of G, for some appropriate expression γ, for some sentence completion β, and for some world $w \in C$,

$$w \not\models \alpha(d \ and \ \gamma)\beta \Leftrightarrow \alpha\gamma\beta.$$

But if so, $w \not\models (not \ \alpha(p \ and \ \gamma)\beta) \Leftrightarrow (not \ \alpha\gamma\beta)]$, and hence not $\mathrm{Transp}(C, F)$. Contradiction.

For the converse, suppose that $C[F] \neq \#$. Then $C[G] \neq \#$, and by the Induction Hypothesis $\mathrm{Transp}(C, G)$. Now suppose, for contradiction, that not $\mathrm{Transp}(C, F)$. Then for some initial string $\alpha\underline{dd'}$ of G, for some appropriate expression γ, for some sentence completion β, and for some $w \in C$,

$$w \not\models (not \ \alpha(d \ and \ \gamma)\beta \Leftrightarrow (not \ \alpha\gamma\beta.$$

By clause (b) of the Syntactic Lemma in (5), $(not \ \alpha(d \ and \ \gamma)\beta$ is the smallest initial string of itself which is a constituent. It follows that β is of the form $\delta)$, and thus:

$$w \not\models (not \ \alpha(d \ and \ \gamma)\delta) \Leftrightarrow (not \ \alpha\gamma\delta)$$

and, therefore,

$$w \not\models \alpha(d \ and \ \gamma)\delta \Leftrightarrow \alpha\gamma\delta.$$

But this shows that not $\mathrm{Transp}(C, G)$. Contradiction.

(ii) If $C[F] \neq \#$, $C[F] = C \setminus C[G]$. By the Induction Hypothesis, $C[G] = \{w \in C : w \models G\}$ and thus $C[F] = C \setminus \{w \in C : w \models G\} = \{w \in C : w \models (not \ G)\}$.

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[3] In fact, the syntax in **2.1** guarantees that the only acceptable sentence completion is one in which β is the null string.

d. $F = (G \ and \ H)$.

(i) Suppose that $\text{Transp}(C, F)$. By the Transparency Lemma (part (a)), $\text{Transp}(C, G)$. By the Induction Hypothesis, $C[G] \neq \#$, and by the Induction Hypothesis (part (ii)) $C[G] = \{w \in C : w \models G\}$. Calling $C' = \{w \in C : w \models G\}$, we claim that $\text{Transp}(C', H)$. For suppose this were not the case. For some initial segment $\alpha \underline{d}d'$ of H, for some appropriate expression γ, for some sentence completion β, and for some world $w' \in C'$, we would have $w' \not\models \alpha(p \ and \ \gamma)\beta \Leftrightarrow \alpha\gamma\beta$. But then w' would refute $\text{Transp}(C, (G \ and \ H))$ because we would have $w' \not\models (G \ and \ \alpha(d \ and \ \gamma)\beta) \Leftrightarrow (G \ and \ \alpha\gamma\beta)$ with $w' \models G$. So $\text{Transp}(C', H)$, and thus by the Induction Hypothesis (Part (i)) $C'[H] \neq \#$, i.e., $C[G][H] \neq \#$. For the converse, suppose that $C[F] \neq \#$. Then $C[G] \neq \#$ and $C[G][H] \neq \#$. By the Induction Hypothesis, $\text{Transp}(C, G)$, $C[G] = \{w \in C : w \models G\}$ (a set we call C'), and $\text{Transp}(C', H)$. Suppose, for contradiction, that not $\text{Transp}(C, F)$, and let $w \in C$ satisfy $w \not\models \alpha(p \ and \ \gamma)\beta \Leftrightarrow \alpha\gamma\beta$, where $\alpha\underline{d}d'$ is an initial string of $(G \ and \ H)$.

– Let us first show that this occurrence of $\underline{d}d'$ is not part of G. For suppose, for contradiction, that it is. Then for some initial string α' of G we have $w \not\models (\alpha'(d \ and \ \gamma)\beta \Leftrightarrow (\alpha'\gamma\beta$. $\alpha'\underline{d}d'$ is the beginning of a constituent in G, and thus by the Syntactic Lemma (part (a)), it is the beginning of a constituent in $(\alpha'\underline{d}d'\beta$. Let β' be the smallest initial string of β for which $\alpha'\underline{d}d'\beta'$ is a constituent. Since $w \not\models (\alpha'(d \ and \ \gamma)\beta \Leftrightarrow (\alpha'\gamma\beta$, it must also be that $w \not\models \alpha'(d \ and \ \gamma)\beta' \Leftrightarrow \alpha'\gamma\beta'$. But this shows that not $\text{Transp}(C, G)$, contrary to what was shown earlier.

– So this occurrence of $\underline{d}d'$ appears in H. Thus for some initial string $\alpha'\underline{d}d'$ of H, for some appropriate expression γ and for some sentence completion β, we have:

$$w \not\models (G \ and \ \alpha'(d \ and \ \gamma)\beta \Leftrightarrow (G \ and \ \alpha'\gamma\beta.$$

Since $\alpha'\underline{d}d'$ is the beginning of a constituent in H, $\alpha'\underline{d}d'$ is also the beginning of a constituent in $\alpha'\underline{d}d'\beta$ (Syntactic Lemma, part (a)). Furthermore, since G is a constituent, $(G \ and \ \alpha'(d \ and \ \gamma)\beta$ and $(G \ and \ \alpha'\gamma\beta$ must be of the form $(G \ and \ \alpha'(d \ and \ \gamma)\beta')$ and $(G \ and \ \alpha'\gamma\beta')$, respectively. It follows that G must be true at w, for otherwise both formulas would be false and they would thus have the same value at w, contrary to hypothesis. So $w \models G$. But since $w \not\models (G \ and \ \alpha'(d \ and \ \gamma)\beta') \Leftrightarrow (G \ and \ \alpha'\gamma\beta')$, so it must be that $w \not\models \alpha'(d \ and \ \gamma)\beta' \Leftrightarrow \alpha'\gamma\beta'$. But then it follows that not $\text{Transp}(C', H)$, since $w \in C'$ and $\alpha'\underline{d}d'$ is an initial segment of H. But this contradicts our hypothesis. Thus $\text{Transp}(C, (G \ and \ H))$, i.e., $\text{Transp}(C, F)$.

(ii) If $C[F] \neq \#$, $C[F] = C[G][H] = \{w \in C : w \models G\}[H] = \{w \in C : w \models (G \ and \ H)\}$.

e. $F = (G \ or \ H)$.

(i) Suppose that $\text{Transp}(C, F)$. Then, by the Transparency Lemma (part (a)), it is also the case that $\text{Transp}(C, G)$. By the Induction Hypothesis, $C[G] \neq \#$, and $C[G] = \{w \in C : w \models G\}$. Therefore $C[(not \ G)] = C \setminus C[G] = \{w \in C : w \not\models G\}$ (call this set C'). It follows that $\text{Transp}(C', H)$, because otherwise for some initial

segment $\alpha\underline{d}d'$ of H, for some appropriate expression γ, for some sentence completion β and for some $w' \in C'$, we would have:

$$w' \not\models \alpha(d \ and \ \gamma)\beta \Leftrightarrow \alpha\gamma\beta.$$

But since $w' \not\models G$,

$$w' \not\models (G \ or \ \alpha(d \ and \ \gamma)\beta) \Leftrightarrow (G \ or \ \alpha\gamma\beta),$$

and thus not $\mathrm{Transp}(C, (G \ or \ H))$, contrary to hypothesis. So $\mathrm{Transp}(C', H)$ and, by the induction hypothesis, $C'[H] \neq \#$, i.e., $C[(not \ G)][H] \neq \#$. By the dynamic semantics of *or*, $C[(G \ or \ H)] \neq \#$. For the converse, suppose that $C[(G \ or \ H)] \neq \#$. Thus $C[G] \neq \#$ and $C[(not \ G)][H] \neq \#$. By the Induction Hypothesis, $\mathrm{Transp}(C, G)$ and $\mathrm{Transp}(C', H)$ with $C' = C[(not \ G)]$. Suppose, for contradiction, that not $\mathrm{Transp}(C, F)$, and let $w \in C$ satisfy $w \not\models \alpha(d \ and \ \gamma)\beta \Leftrightarrow \alpha\gamma\beta$, where $\alpha\underline{d}d'$ is an initial string of $(G \ or \ H)$.

– Let us first show that this occurrence of $\underline{d}d'$ is not part of G. Suppose, for contradiction, that it is. Then for some initial string $\alpha'\underline{d}d'$ of G, for some appropriate expression γ, for some sentence completion β and for some $w \in C$, we have

$$w \not\models (\alpha'(d \ and \ \gamma)\beta \Leftrightarrow (\alpha'\gamma\beta.$$

$\alpha'\underline{d}d'$ is the beginning of a constituent in G, and thus, by the Syntactic Lemma (part (a)), it is the beginning of a constituent in $(\alpha'\underline{d}d'\beta$. Let β' be the smallest initial string of β for which $\alpha'\underline{d}d'\beta'$ is a constituent. Since $w \not\models (\alpha'(p \ and \ \gamma)\beta \Leftrightarrow (\alpha'\gamma\beta$, it must also be that $w \not\models \alpha'(p \ and \ \gamma)\beta' \Leftrightarrow \alpha'\gamma\beta'$. But this shows that not $\mathrm{Transp}(C, G)$, contrary to what was shown earlier.

– So this occurrence of $\underline{d}d'$ appears in H. Thus for some initial string $\alpha'\underline{d}d'$ of H, for some appropriate expression γ, for some sentence completion β and for some $w \in C$ we have:

$$w \not\models (G \ or \ \alpha'(p \ and \ \gamma)\beta \Leftrightarrow (G \ or \ \alpha'\gamma\beta.$$

Since $\alpha'\underline{d}d'$ is the beginning of a constituent in H, $\alpha'\underline{d}d'$ is also the beginning of a constituent in $\alpha'\underline{d}d'\beta$ (Syntactic Lemma, part (a)). Furthermore, since G is a constituent, $(G \ or \ \alpha'(d \ and \ \gamma)\beta$ and $(G \ or \ \alpha'\gamma\beta$ must be of the form $(G \ or \ \alpha'(d \ and \ \gamma)\beta')$ and $(G \ or \ \alpha'\gamma\beta')$, respectively. It follows that G must be false at w, for otherwise both formulas would be true and they would thus have the same value at w, contrary to hypothesis. So $w \not\models G$. But since $w \not\models (G \ or \ \alpha'(d \ and \ \gamma)\beta') \Leftrightarrow (G \ or \ \alpha'\gamma\beta')]$, it must be that $w \not\models \alpha'(d \ and \ \gamma)\beta' \Leftrightarrow \alpha'\gamma\beta'$. But then it follows that not $\mathrm{Transp}(C', H)$, since $w \in C'$ and $\alpha'\underline{p}p'$ is an initial segment of H. But this contradicts our hypothesis that $\mathrm{Transp}(C', H)$. Thus $\mathrm{Transp}(C, (G \ and \ H))$, i.e., $\mathrm{Transp}(C, F)$.

(ii) If $C[F] \neq \#$, then $C[G] \neq \#$, $C[(not \ G)][H] \neq \#$, and $C[F] = C[G] \cup C[(not \ G)][H]$. By the Induction Hypothesis, $C[G] = \{w \in C : w \models G\}$, $C[(not \ G)] = \{w \in C : w \not\models G\}$, and $C[(not \ G)][H] = \{w \in C : w \not\models G \wedge w \models H\}$. Therefore, $C[F] = \{w \in C : w \models G\} \cup \{w \in C : w \not\models G \wedge w \models H\} = \{w \in C : w \models (G \ or \ H)\}$.

f. $F = (if\ G.\ H)$.

(i) Suppose $\mathrm{Transp}(C, F)$. By the Transparency Lemma (part (b)), it must also be the case that $\mathrm{Transp}(C, G)$. Let us now show that $\mathrm{Transp}(C', H)$ with $C' = C[G]$. Suppose, for contradiction, that this is not the case. Then for some initial segment $\alpha \underline{dd}'$ of H, for some appropriate expression γ, for some sentence completion β, and for some $w' \in C'$,

$$w' \not\models \alpha(d\ and\ \gamma)\beta \Leftrightarrow \alpha\gamma\beta.$$

Since $w' \in C'$, it must also be the case that

$$w' \not\models (if\ G.\ \alpha(d\ and\ \gamma)\beta) \Leftrightarrow (if\ G.\ \alpha\gamma\beta),$$

hence not $\mathrm{Transp}(C, (if\ G.\ H))$, contrary to hypothesis. So $\mathrm{Transp}(C', H)$, and thus $C[G][H] \neq \#$. Since $C[G] \neq \#$, and $C[G][H] \neq \#$, $C[(if\ G.\ H)] \neq \#$. For the converse, let us assume that $C[F] \neq \#$. Then $C[G] \neq \#$ and $C[G][H] \neq \#$. Hence $\mathrm{Transp}(C, G)$ and $\mathrm{Transp}(C', H)$ with $C' = C[G]$, from which it also follows that $\mathrm{Transp}(C', not\ H)$.

Now suppose, for contradiction, that not $\mathrm{Transp}(C, (if\ G.\ H))$. Then for some initial segment $\alpha \underline{dd}'$ of $G.H$, for some appropriate expression γ, for some sentence completion β, and for some $w \in C$, we have

$$w \not\models (if\ \alpha(d\ and\ \gamma)\beta \Leftrightarrow (if\ \alpha\gamma\beta.$$

- It could not be the case that this occurrence of $\underline{dd}'$ is in G, because in that case we would have for some strings β' and β'':

$$w \not\models (if\ \alpha'(d\ and\ \gamma)\beta'.\ \beta'' \Leftrightarrow (if\ \alpha'\gamma\beta'.\ \beta'',$$

which could only be the case if $w \not\models \alpha'(d\ and\ \gamma)\beta' \Leftrightarrow \alpha'\gamma\beta'$, and hence if not $\mathrm{Transp}(C, G)$, contrary to what we showed earlier.

- Now suppose that this occurrence of $\underline{dd}'$ is in H. For some initial string $\alpha\underline{dd}'$ of H, for some appropriate expression γ, for some sentence completion β, and for some $w \in C$, we have

$$w \not\models (if\ G.\ \alpha(d\ and\ \gamma)\beta) \Leftrightarrow (if\ G.\ \alpha\gamma\beta).$$

But then it must also be the case that $w \models G$, for otherwise both sides of the biconditional would be true at w. Furthermore, it must be the case that $w \not\models \alpha(d\ and\ \gamma) \Leftrightarrow \alpha\gamma\beta$, because otherwise we would have

$$w \models (if\ G.\ \alpha(d\ and\ \gamma)\beta) \Leftrightarrow (if\ G.\ \alpha\gamma\beta).$$

But this shows that not $\mathrm{Transp}(C', H)$, contrary to what we showed earlier. In sum, $\mathrm{Transp}(C, F)$.

(ii) If $C[F] \neq \#$, $C[F] = C \setminus C[G][not\ H]$. But, by the Induction Hypothesis, $C[G] = \{w \in C : w \models G\}$ and $C[G][not\ H] = \{w \in C : w \models G\}[not\ H] = \{w \in C : w \models (G\ and\ (not\ H))\}$, and thus $C[F] = \{w \in C : w \not\models (G\ and\ (not\ H))\} = \{w \in C : w \models (if\ G.\ H)\}$.

4 Quantificational case

We now turn to the quantificational case, which we treat separately because it involves additional complications and leads to weaker equivalence results than the propositional case. Heim's claim is that for any generalized quantifier Q,

(i) $(Q\underline{P}P'.R)$ presupposes that every individual in the domain satisfies P, and

(ii) $(QP.\underline{R}R')$ presupposes that every individual in the domain that satisfies P also satisfies R.[4]

We will find conditions under which these predictions are indeed derived from our system. We start by stating the conditions, and then we construct the proof in two steps: first, we obtain the desired result for quantificational formulas that are unembedded; second, we integrate the argument into a proof by induction that extends to all formulas of the language.

4.1 Non-triviality and constancy

The equivalence with Heim's result turns out to be weaker than in the propositional case; it holds only when the Context Set satisfies additional constraints. To see why a weaker result is obtained, let us note that there could be a world w in which Transparency is satisfied because Q has a degenerate semantics. Consider the following scenario:

- In w, there are exactly 2 P-individuals, one of whom satisfies R and one of whom does not.

- The sentence uttered is $(QP.\underline{R}R')$ with $Q = less\ than\ three$.

Even though it is not the case that each P-individual satisfies R in w, Transparency is trivially satisfied with respect to w, because for any predicative expression γ,

$$w \models (QP.(R\ and\ \gamma)) \Leftrightarrow (QP.\gamma).$$

Of course, the equivalence holds because *no matter what the nuclear scope Y is, $(QP.Y)$* is true in w: since there are exactly two P-individuals, *a fortiori* there are less than three individuals that satisfy both P and Y.

We will solve the problem by making two assumptions:

(i) First, we require that each quantificational clause should make a non-trivial contribution to the truth conditions (= Non-Triviality). Specifically, we require that for each initial string αA of any sentence uttered in a Context Set C, where A is a quantificational clause (i.e., a clause of the form $(Q_i G.H)$), there is at least one sentence completion β for which A makes a semantic contribution that could not be obtained by replacing A with a tautology T or a contradiction F. Thus Non-Triviality requires that for some sentence completion β,

$$C \not\models \alpha A\beta \Leftrightarrow \alpha T\beta;$$
$$C \not\models \alpha A\beta \Leftrightarrow \alpha F\beta.$$

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[4] Heim (1983) observes that special provisions are needed for indefinites, which trigger extremely weak presuppositions. Thus *A fat man was pushing his bicycle* certainly doesn't presuppose that every fat man had a bicycle. We disregard this point in what follows (see Schlenker 2006b for a remark on the treatment of indefinites in the Transparency framework).

If the Context Set only includes worlds with less than three P-individuals, Non-Triviality will automatically rule out any sentence of the form $\alpha(QP.\underline{R}R')\beta$ for $Q = \textit{less than three}$. This is because when one has heard $\alpha(QP.\underline{R}R')$, one can determine that one can replace $(QP.\underline{R}R')$ with T without modifying the contextual meaning of the sentence, no matter how it ends.

(ii) This measure won't be enough, however. Suppose that $C = \{w, w', w''\}$, where w is the world mentioned earlier in which there are exactly two P-individuals, while w' and w'' are worlds that have exactly four P-individuals, with the following specifications:

w': all P-individuals satisfy R and R'.
w'': all P-individuals satisfy R but no P-individual satisfies R'.

Consider the sentence $(QP.\underline{R}R')$. As before, Transparency is satisfied in w (despite the fact that in w some P-individual does not satisfy R). Furthermore, Transparency is also satisfied in w' and w'', because in these worlds each P-individual satisfies R. Contrary to the case we considered in (i), however, *this situation is not ruled out by Non-Triviality:*

$w' \not\models (QP.\underline{R}R') \Leftrightarrow T$ (the left-hand side is false, but the right-hand side is true);
$w'' \not\models (QP.\underline{R}R') \Leftrightarrow F$ (the left-hand side is true, but the right-hand side is false).

In this counter-example, however, it is crucial that the extension of P does not have the same size in w ($|P^w| = 2$) and in w' and w'' ($|P^{w'}| = |P^{w''}| = 4$). We will see that this property is indeed essential to construct the problematic examples, and that when Non-Triviality is combined with the requirement ('Constancy') that the size of the extension of each restrictor be fixed throughout the Context Set, the equivalence with Heim's theory can indeed be achieved.

Before we prove our (limited) equivalence result, let us give a precise statement of Non-Triviality:

(10) **Definition of Non-Triviality** Let C be a Context Set, and let F be a formula. $\langle C, F \rangle$ satisfies Non-Triviality just in case for any initial string of the form αA, where A is a quantificational clause (i.e., a formula of the form $(Q_i G.H)$), there is a sentence completion β such that:

$$C \not\models \alpha A\beta \Leftrightarrow \alpha T\beta;$$
$$C \not\models \alpha A\beta \Leftrightarrow \alpha F\beta,$$

where T is a tautology and F is a contradiction.

An immediate consequence of the definition will turn out to be useful:

(11) **Non-Triviality Corollary** Let Q_i be a generalized quantifier with the associated tree of numbers f_i. Consider a formula $(Q_i G.H)$, evaluated in a Context Set C. Then:

(i) If $\langle C, (Q_i G.H) \rangle$ satisfies Non-Triviality and if in C the domain of individuals is of constant finite size n, then

$$\{f_i(a, b): a, b \in \mathbb{N} \wedge a + b \leq n\} = \{0, 1\};$$

(ii) If $\langle C, (Q_i G.H) \rangle$ satisfies Non-Triviality and if in C the extension of G is of constant finite size g, then

$$\{f_i(a, b): a, b \in \mathbb{N} \wedge a + b = g\} = \{0, 1\}.$$

4.2 Sketch of the proof

As announced, the proof proceeds in two steps. First, we show that under the assumptions of Constancy and Non-Triviality, the Transparency framework makes the same predictions as Heim's system for unembedded quantificational sentences. Second, we integrate this result to a proof by induction that applies to all sentences of the language.

Lemma 1

Let Q_i be a generalized quantifier with the associated tree of numbers f_i.

(i) Suppose that

   (a) throughout C, the domain of individuals is of constant finite size n;
   (b) any property over the domain can be expressed by some predicate; and
   (c) $\{f_i(a,b): a, b \in \mathbb{N} \land a + b \leq n\} = \{0, 1\}$.

   Then $\mathrm{Transp}(C, (Q_i\underline{P}P'.R))$ iff $C \models \forall d P(d)$.

(ii) Suppose that

   (a) throughout C, the extension of P is of constant finite size p;
   (b) any property over the domain can be expressed by some predicate; and
   (c) $\{f_i(a,b): a, b \in \mathbb{N} \land a + b = p\} = \{0, 1\}$.

   Then $\mathrm{Transp}(C, (Q_iP.\underline{R}R'))$ iff $C \models \forall d[P(d) \Rightarrow R(d)]$.

Remark: By the Non-Triviality Corollary:
   (i.c) can be replaced with '$\langle C, (Q_i\underline{P}P'.R)\rangle$ *satisfies Non-Triviality*', and
   (ii.c) can be replaced with '$\langle C, (Q_iP.\underline{R}R')\rangle$ *satisfies Non-Triviality.*'
Proof: Omitted for brevity. We note that in (i) and (ii) the *if* part is immediate (for (ii), because of Conservativity), and thus only the *only if* part needs to be discussed. See the full paper for a proof, in which crucial use is made of the fact that Q_i can be represented in terms of the tree of numbers $f_i(a,b)$, for variable a and b.

We must now combine Lemma 1 with the equivalence proof developed for the propositional case to yield a result that holds of quantificational languages. We will do so in two steps:

(i) First, we show, in Lemma 2, that if $\langle C, F\rangle$ satisfies Non-Triviality, then all the pairs $\langle C', F''\rangle$ which must be 'accessed' (in a sense to be made precise) in the computation of $C[F]$ also satisfy Non-Triviality.

(ii) Second, we combine the results of Lemma 1 and Lemma 2 to provide a general equivalence result between Transparency and Heim's results for quantificational languages.

We start by defining the pairs $\langle C, F\rangle$ which must be 'accessed' in the computation of $C[F]$.

Definition 1

Let C be a Context Set, and F be a formula. We simultaneously define the relation $\langle C', F'\rangle$ **is accessed by** $\langle C, F\rangle$ and $\langle C'', F''\rangle$ **is a parent of** $\langle C', F'\rangle$ by the following induction:

(i) $\langle C, F \rangle$ is accessed by $\langle C, F \rangle$;

(ii) If $\langle C', (not\ F') \rangle$ is accessed by $\langle C, F \rangle$, then $\langle C', F' \rangle$ is accessed by $\langle C, F \rangle$ and $\langle C', (not\ F') \rangle$ is the parent of $\langle C', F' \rangle$.

(iii) If $\langle C', (G\ and\ H) \rangle$ is accessed by $\langle C, F \rangle$, then $\langle C', G \rangle$ is accessed by $\langle C, F \rangle$ and $\langle C', (G\ and\ H) \rangle$ is the parent of $\langle C', G \rangle$; and if $C'[G]$ is defined, $\langle C'[G], H \rangle$ is accessed by $\langle C, F \rangle$ and $\langle C', (G\ and\ H) \rangle$ is the parent of $\langle C'[G], H \rangle$.

(iv) If $\langle C', (G\ or\ H) \rangle$ is accessed by $\langle C, F \rangle$, then $\langle C', G \rangle$ is accessed by $\langle C, F \rangle$ and $\langle C', (G\ or\ H) \rangle$ is the parent of $\langle C', G \rangle$; and if $C'[G]$ is defined, $\langle C'[(not\ G)], H \rangle$ is accessed by $\langle C, F \rangle$ and $\langle C', (G\ or\ H) \rangle$ is the parent of $\langle C'[(not\ G)], H \rangle$.

(v) If $\langle C', (if\ G.\ H) \rangle$ is accessed by $\langle C, F \rangle$, then $\langle C', G \rangle$ is accessed by $\langle C, F \rangle$ and $\langle C', (if\ G.\ H) \rangle$ is the parent of $\langle C', G \rangle$; and if $C'[G]$ is defined, $\langle C'[G], H \rangle$ is accessed by $\langle C, F \rangle$ and $\langle C', (if\ G.\ H) \rangle$ is the parent of $\langle C'[G], H \rangle$.

Lemma 2

Suppose that $\langle C, F \rangle$ satisfies Non-Triviality. Then if $\langle C', F' \rangle$ is accessed by $\langle C, F \rangle$, $\langle C', F' \rangle$ satisfies Non-Triviality as well.

Proof: One shows by induction that if $\langle C', F' \rangle$ is accessed by $\langle C, F \rangle$ and violates Non-Triviality, then either $\langle C', F' \rangle = \langle C, F \rangle$, or $\langle C', F' \rangle$ has a parent that also violates Non-Triviality. A trivial induction on the definition of pairs $\langle C', F' \rangle$ that are accessed by $\langle C, F \rangle$ will then yield the Lemma.

Theorem 2

Let C be a Context Set and F be a formula. Suppose that (i) the domain of individuals is of constant size over C; (ii) the extension of each restrictor that appears in F is of constant size over C; and (iii) $\langle C, F \rangle$ satisfies Non-Triviality. Then for every $\langle C', F' \rangle$ which is accessed by $\langle C, F \rangle$ (including $\langle C, F \rangle$ itself):

(i) $\mathrm{Transp}(C', F')$ iff $C'[F'] \neq \#$.

(ii) If $C'[F'] \neq \#$, $C'[F'] = \{w \in C' : w \models F'\}$.

Proof: Omitted for brevity. The argument is by induction on the construction of F'. It is similar to the proof of Theorem 1, with some additions to steps (a–f) and one additional step (for the quantificational case).

REFERENCES

Beaver, D. I. 2001. Presupposition and Assertion in Dynamic Semantics. Studies in Logic, Language and Information, Stanford CA: CSLI Publications.

van Benthem, J. 1986. Essays in Logical Semantics. Dordrecht: Reidel.

Davis, S. (ed.) . 1991. Pragmatics: A Reader. Oxford University Press.

Heim, I. 1983. On the projection problem for presuppositions. In: D. Flickinger *et al.* (ed.). Proceedings of the Second West Coast Conference on Formal Linguistics. 114–125. Reprinted in Davis 1991.

Heim, I. 1990. Presupposition projection. In: R. van der Sandt (ed.). Reader for the Nijmegen Workshop on Presupposition, Lexical Meaning, and Discourse Processes. Nijmegen: University of Nijmegen.

Heim, I. 1992. Presupposition projection and the semantics of attitude verbs. Journal of Semantics 9: 183–221.

Keenan, E. 1996. The semantics of determiners. In: S. Lappin (ed.). The Handbook of Contemporary Semantic Theory. Blackwell.

Schlenker, P. 2006a. Anti-dynamics: Presupposition projection without dynamic semantics. Preliminary draft, `http://www.linguistics.ucla.edu/people/schlenker/Anti-Dynamics.pdf`.

Schlenker, P. 2006b. Transparency: An incremental theory of presupposition projection. In: U. Sauerland and P. Stateva (eds.). Presupposition and Implicature in Compositional Semantics. Basingstoke UK: Palgrave Macmillan. To appear.

Schlenker, P. 2006c. Transparency: the projection problem revisited. 2-page abstract for the Milan Meeting.

Soames, S. 1989. Presupposition. In: D. Gabbay and F. Guenthner (eds.). Handbook of Philosophical Logic. Kluwer, volume 4. 553–616.