

Some modifiers of conditionals

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1 Introduction

Conditional sentences are difficult to analyse and the literature on this topic is so abundant that I do not dare to mention any title except for those mentioned for the specific purposes of this paper. The difficulty starts already at classificational and typological levels. The purpose of this paper is to analyse conditional sentences (CSs, for short) in which specific items, so-called *categorially polyvalent particles*, CPPs for short, occur. CPPs are functional expressions which can have as their possible arguments expressions of different grammatical categories. Grammatically such expressions are usually *modifiers*, that is functional expressions of the category C/C for various categories C . The interest of CPPs is that in spite of the fact that they can apply to expressions of various categories (in that sense they are *categorially polyvalent*) they usually have in some sense a constant meaning across categories. So it might be interesting to see whether this constancy of meaning is preserved in the context of CSs as well.

My analysis will be carried in the framework of Boolean semantics and the reason is that it is precisely in this framework that the notion of categorial polyvalency and of categorially polyvalent modification get their clear meaning.

The classical cases of such categorially polyvalent modifiers are items like *only*, *also* and *even* as shown in the following examples:

- (1)
 - a. (Only/also/even Leo) danced on weekdays with Lea in the garden.
 - b. Leo (only/also/even danced) on weekdays with Lea in the garden.
 - c. Leo danced (only/also/even on weekdays) with Lea in the garden.
 - d. Leo danced on weekdays (only/also/even with Lea) in the garden.
 - e. Leo danced on weekdays with Lea (only/also/even in the garden).
- (2) Leo danced on weekdays with Lea in the garden.

The above examples show that classical CPPs are categorially polyvalent modifiers: they can apply to expressions of various categories and the resulting expression is of the same category as the argument expression. For instance in (1a) they apply to NPs and give as a result NPs, in (1b) they modify verbs, in (1c) they modify adverbials and so on. From the semantic point of view we observe for the moment that they denote restrictive functions in the sense that they complex unit resulting from their application entails the modified argument. In the above case all sentences in (1) entail (2).

Of course, as we will see in some detail, classical CPPs can also modify CSs. Various analyses of CSs modified by some classical CPPs, in particular by *even*, have been proposed (Abott 2005; Lycan 1991; Berckmans 1993). We will see, and this is an empirical contribution of this paper, that there are many other categorially polyvalent CPPs which also can modify CSs. Furthermore, it will be shown that classical CPPs, at least *only* and *even*, are logically basic in the sense that many other ‘non-classical’ CPPs can be obtained from classical ones by Boolean operations.

2 Other cases

In this section we present some other, less often discussed and probably less known, cases of categorially polyvalent modifiers. We will see that they also can modify CSs and some of them will be analysed in this context . First notice the following examples given here just as an illustration:

- (3) a. Some teachers, in particular/especially Leo, think that ...
- b. Yesterday he did many things, in particular he finished his paper.
- c. He sings everywhere, in particular in his bathroom.
- (4) a. Leo will not come, let alone Lea.
- b. Leo does not work on Saturdays, let alone on Sundays.
- c. Leo does not smoke, let alone drink.

Surprisingly, *at least*, *at most* usually considered as modifiers of numerals are in fact categorially polyvalent modifiers:

- (5) a. At least/at most Lea will pass the examination.
- b. Lea sings at least/at most in the bathroom.
- c. At least/at most five teachers were there.
- d. At most/at least he can walk.

In (5a) we have a modification of an NP by *at least*, *at most* , in (5b) these items modify adverbials and in (5c) they modify a VP.

Many CPPs occur in CSs. The cases *only*, *also* and *even* are well-known (cf. *only if*, *also if* and *even if* . In (6c) we have a modification of an *if*-clause by *at least* and (6d) shows that such a modification by *at most* is impossible:

- (6) a. Lea will be happy, in particular if Leo calls.
- b. Lea will not be happy if it rains, let alone if it snows.
- c. Lea will call, at least if it rains.
- d. *Lea will call, at most if it rains.

There are similarities and differences between various CPPs and they should be taken into account in the final an analysis of these particles considered as categorially polyvalent. One observes for instance *only*, *also* and *even*, *at least* and *at most* need not occur with additional lexical material when applying to a particular argument. This does not seem to be the case with particles like *especially* and *in particular*. Furthermore, there is a systematic semantic relationship between the additional lexical material and the argument of these particles suggesting that the explicitly required lexical material plays a role of an anaphora-antecedent like element:

- (7) a. *In particular/especially Leo will call.
- b. Some students, in particular Leo, will call.
- c. Some students and in particular Leo, will call.
- d. *He sings in his office, in particular/especially in the bathroom.
- (8) a. He sings everywhere, in particular in his bathroom.
- b. He sings in his office and in particular in his bathroom.
- c. *He likes wine, in particular chocolate.

- d. He likes wine, in particular champagne.
- e. He likes wine and in particular chocolate.

In addition to these differences there are also similarities concerning the semantic contribution of some CPPs independently on whether they apply to conditional or non-conditional arguments. One of them concerns the so-called consequent entailment problem in *even if* conditionals. It has been claimed that *even if* conditionals should not be considered semantically as conditionals since such conditionals assert the truth of the consequent unconditionally: the speaker who asserts *P even if Q* seems to be asserting that *P* holds independently of *Q*. For instance (9a) seems to entail (9b):

- (9) a. Leo will leave even if he is tired.
- b. Leo will leave.

Notice now that something similar happens in the case when *even* modifies arguments of other categories. For instance (10a) seems to entail (10b):

- (10) a. Even Leo danced.
- b. Everybody danced.

The similarity is obvious: in both cases, in (9) and in (10), there seems to be a hidden universal quantifier which in some way imposes an unrestricted reading.

The data are a bit more complex, however. Consider the following examples:

- (11) Leo will leave if it rains and even if he is tired.
- (12) Lea and even Leo danced.

The supposed entailment we had in (9) or in (10) does not hold anymore in (11) and (12). Thus the truth of (11) does not force us to consider (9b) as true and (12) does not entail (10b).

The above examples show that a complete analysis of CPPs should take into account differences and similarities between various CCPs. In this paper I am basically interested in similarities between various particles and I show that it is possible to analyse CPPs in an uniform way using algebraic tools of the Boolean semantics.

3 Boolean semantics

Boolean semantics (Keenan & Faltz 1985) is a version of formal semantics which explicitly assumes that the semantic types have Boolean structure. Thus for any category *C* there is a corresponding denotational Boolean algebra D_C of possible denotations of expressions of category *C*. The algebra $D_{A/B}$ has as elements functions from D_B to D_A . D_C are atomic. Atoms of the algebra $D_{A/B}$ are determined by atoms and/or elements of the resulting algebra D_A .

We are interested here basically in the denotational algebras of modifiers. A modifier is a functional expression of category C/C for various choices of *C*. Modifiers of category C/C denote in the denotational algebra of restrictive functions $\text{RESTR}(C)$, which is a subset of the set of functions from D_C onto D_C . The set $\text{RESTR}(C)$ of restrictive functions $f_c \in D_{C/C}$, is the set of functions satisfying the condition $f_c(x) \leq x$, for any $x \in D_C$ (Keenan & Faltz 1985). The set of restrictive functions forms a Boolean algebra:

Proposition 1 *Let B be a Boolean algebra. Then the set of functions f from B onto B satisfying the condition $f(x) \leq x$ forms a Boolean algebra $\text{RESTR}(B)$ with the Boolean*

operations of meet and join defined pointwise and where $0_{RESTR(B)} = 0_B$, $1_{RESTR(B)} = id_B$, $f'(x) = x \cap (f(x))'$.

Proposition 1 shows how to form the restrictive Boolean algebra $RESTR(B)$ from the algebra B . What is important here is the fact that the Boolean complement is relativised to the one element of the algebra which is just the identity function.

Restrictive algebras are also atomic:

Proposition 2 *If B is atomic so is $RESTR(B)$. For all $b \in B$ and all atoms α of B such that $\alpha \leq b$, functions $f_{b,\alpha}$ defined by $f_{b,\alpha}(x) = \alpha$ if $x = b$ and $f_{b,\alpha}(x) = 0_B$ if $x \neq b$ are the atoms.*

There is an important sub-class $ABS(B)$ of restrictive functions (relative to a given Boolean algebra B): these are the so-called *absolute functions*. By definition $f \in ABS(B)$ iff for any $x \in B$, we have $f(x) = x \cap f(1_B)$. One can show that $ABS(B)$ is a sub-algebra of R_B . The atoms and co-atoms of $ABS(B)$ are indicated in:

Proposition 3 *If B is atomic so is $ABS(B)$. For all atoms α of B , functions f_α , defined by $f_\alpha(x) = \alpha \cap x$ are the atoms of $ABS(B)$. For all atoms α of B , functions $f_{\alpha'}$, defined by $f_{\alpha'}(x) = x \cap \alpha'$ are the co-atoms of $ABS(B)$.*

Atoms of both algebras, the algebra R_B and the algebra $ABS(B)$ will be used to interpret CPPs. The algebra B corresponds to the denotational algebra of the argument to which the CPP applies.

4 The meaning of CPPs

How it is possible that CPPs keep their general meaning constant across categories. I propose to explain this meaning constancy of CPPs across categories by relating their denotations to atomicity of corresponding denotational algebras. Thus, in the simplest case an expression with a CPP denotes an atom in the algebra whose type is determined by the category of the argument of the particle. Other particles denote Boolean combinations of atoms and, possibly, of ‘variables’ of appropriate category. For instance expressions denoting co-atoms, that is Boolean complements of atoms, can also be considered as having a general, category independent meaning given that Boolean complements have such a meaning as well. Similarly a function of the form $f_c(x_c) = x_c \vee_c at_c$, can be considered as having a general meaning independent of category c because in its definition category independent operations are used.

Let us consider first the classical CPPs *only*, *also* and *even*. We observe that all these particles are semantically modifiers denoting restrictive functions. This means in particular that the sentences with a particle entail the corresponding ‘particle-less’ sentence. Their meaning constancy is due to the fact that their denotations are linked to atomicity. The case of *only* is relatively easy. We can explain its meaning constancy across categories by saying that *only* always denotes atoms of the denotational algebras of modifiers (Zuber 2001). Which exact atom and in which algebra depends on the category and value of the argument of *only*. Thus *only* in *only NP* denotes an atom in $D_{NP/NP}$, *only* in *only yesterday* denotes an atom in $D_{VP/VP}$, *only* in *only five* denotes an atom in the denotational algebra of modifiers of numerals (or determiners), etc.

This proposal concerning the relationship between *only* and atomicity can be justified more easily for some categories than for others. One can give an ‘almost formal’ proof that

only NP denotes an atom of D_{NP} using the fact that there is an isomorphism between the algebra D_{INT} of intersective determiners and the algebra D_{NP} (Zuber 2001).

Recall that there are at least two types of modifiers, those denoting in restrictive algebras and those denoting in absolute algebras. The particle *only* in its ‘usual’ meaning denotes an atom of an absolute algebra. It is possible, however, that in some uses *only* has also a scalar meaning (for instance in MSCCs) and in this case it denotes an atom of a restrictive algebra.

Let us see now some other particles. There are some arguments (Zuber 2004) showing that *also* is the Boolean complement of *only*:

$$\text{ALSO}(X) = \text{ONLY}'(X).$$

Indeed *not only Leo* cross-categorially entails *also Leo* and *also Leo* cross-categorially entails *not only Leo*.

CPP *even* can be analysed as denoting an atomic function of the algebra of restrictive non-absolute modifiers. As indicated above, such functions are determined by two indices: an element of the denotational algebra of arguments of *even* and an atom included in this element. When the arguments are NPs atoms of the corresponding denotational algebras are singletons containing a property as a unique element. We obtain this property by taking the property corresponding to the VP of the sentence in which the subject NP is modified by *even* and intersecting it with the property pragmatically incompatible with it. There are two arguments for such a move. First, a conjunction of two NPs modified by *even* is impossible: **even Leo and even Lea*. Second, quantified NPs with *even* exhibit quantifier constraint in the same way as exception NPs (which are related to atoms). Thus we do not have **most/*some students, except Leo; *most/*some students, even Leo* but we do have *every student except Leo; every student, even Leo*. Given this (13) can be analyzed as in (14):

(13) Even Leo danced.

(14) $\text{EVEN } L \text{ DANCED} = \text{ONLY } L \text{ IS } D \cap \text{Inc}(D)$

The description in (14) is given in the appropriate metalanguage. Informally it means that Leo is the only dancer who has a property incompatible with dancing. This uniqueness related the meaning of *even* to atomicity and, at the same time, gives rise to the surprise effect usually associated with the meaning of *even*.

Using the above description of classical CPPs we can define the meaning of other CPPs. Thus the meaning of *et least* is given in (15) and the meaning of *at most* is given in (16):

(15) $\text{AT-LEAST}(X) = X \text{ OR NOT-ONLY}(X)$

(16) $\text{AT-MOST}(X) = \text{ONLY}(X) \text{ OR NOT-EVEN}(X)$

Notice that descriptions in (15) and (16) are category (type) independent. This means that the variable X above can be of any (major) category. For instance *at most Leo* ‘means’ ‘Only Leo or not even Leo’.

5 Conditionals

Before extending the above description of CPPs to the case when they modify conditional sentences I need to mention a class of conditional sentences which are excluded from this analysis and which seem to be related to conditionals in which a modification by a CPP

occurs. The conditionals I will not discuss here are so-called *minimal sufficient condition conditionals*, MSCC, that is conditionals which express *minimal sufficient conditions* (cf. Zuber 2006b). Roughly speaking MSCCs are conditional sentences of the form P IF ONLY Q. In other words the conditional connector in this case is the connector IF ONLY (and not ONLY IF).

An English example of MSCC would be *He would be happy if only he had a bottle of wine*. In fact such constructions seem rather restricted in English since apparently IF ONLY clauses are preferably used in English in counterfactuals and in ‘incomplete conditionals’ expressing wishes (as in *If only she were intelligent*). Conditional constructions corresponding to MSCCs are very productive in many other languages, in Slavic languages in particular. In addition in these languages exist temporal MSCCs constructed with ONLY WHEN clauses. Many MSCCs expressed in other languages are not easily translatable into English.

The reason that MSCCs should be considered as conditionals modified by CPPs is not only the use of the connector IF ONLY in MSCCs. One observes in addition that both types of conditionals the ‘adverbial’ *then* cannot occur (importance of this fact for the analysis of ONLY IF conditionals has been noticed in Iatridou n.d.). Furthermore, in Japanese the CCP used in MSCCs is not ONLY but the particle corresponding to EVEN (*sae* in Japanese).

We can now apply the above description of CPPs to analyse conditionals modified by CPPs. Such an application is in principle independent of any particular theory of conditionals, even if a theory of conditionals in the framework of Boolean semantics would be more appropriate. In Zuber (2003) it is shown that conditional sentences have in fact a Boolean structure. In particular it is shown that the conjunction of two conditional clauses, IF P AND IF Q, should not have the same interpretation as the of the conditional operator applied to a conjunction of two sentences (that is it should be different from IF (P and Q)). Furthermore, the if-clause can be interpreted as a modifier of the consequent clause. This modifier can be said to be dual to the restrictive modifiers presented above since, roughly, in this case the argument entails the modified argument.

The extension of the analysis of CPPs applying to non-conditional arguments, as illustrated in particular in examples (13–16), to conditional arguments gives the following results for modified conditional sentences:

(17) $P \text{ ALSO IF } Q = P \text{ NOT-ONLY IF } Q$

(18) $P \text{ AT LEAST IF } Q = P \text{ IF } Q \text{ OR } P \text{ NOT-ONLY IF } Q$

(19) $*P \text{ AT MOST IF } Q = P \text{ ONLY IF } Q \text{ OR } P \text{ NOT EVEN IF } Q$

Notice that the description of conditional sentences with *at most* given in (19) indicates that they are uninformative hence probably their ungrammaticality. Furthermore, concerning *even* it follows from my proposal that *even if* conditionals do not entail their consequent and thus the consequent entailment thesis is false. This is because, as indicated above, (20) does not entail that that Leo will dance (in the same way as (21) does not entail that everybody danced, even when the involved set of participants is contextually restricted):

(20) Leo will dance if it rains and even if it snows.

(21) Leo and even Lea will dance.

As far as I can tell the above results are in agreement with our basic intuition concerning the meaning of conditional sentences in general and conditional sentences modified by CCPs in particular.

6 Conclusions

Using the Boolean semantics and in particular the fact that denotational algebras are atomic we analysed CPPs in an unified way which allows us to understand why such particles keep their meaning constant independently of the category of the argument to which they apply, even if they apply to such complex objects as CSs. This is possible because in the Boolean semantics one can naturally use category (type) independent notions such as Boolean operations and atoms. In this paper an additional attempt has been made to explain the surprise effect induced by some CPPs (*even, in particular*): it is proposed that the surprise effect is due to exceptionality of atomic elements in restrictive (non-absolute) algebras. A full analysis of this problems necessitates additional tools since items inducing the surprise effect seem also to induce intensionality (Zuber 2006a). For instance the following two sentences need not have the same truth-value even if the set of dancers and singers is the same:

- (22) a. Even Leo is dancing.
b. Even Leo is singing.

Similarly the conditional sentences of the form ‘*P* even if/in particular if *Q*’ and ‘*P* even if /in particular if *Q*’ need not have the same truth-value in the case when *Q* and *Q*’ have the same truth-value.

Notice finally that my proposal applies also to various non-declarative conditional sentences:

- (23) Open the window only /even/in particular if it rains.
(24) Will you leave even/also if she stays?

I believe that an analysis of such non-declarative conditional sentences along the lines suggested here will not only tell us something about conditionals but also about the meaning of non-declarative sentences.

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