# Linguistic pathway to multiplication

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Project 108951
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# Research question

**Preschoolers (even infancts) can perform intuitive addition and subtraction –**

**but**

**are children capable of multiplicative operations on sets prior to schooling?**

# Claim:

**Multiplication operations are routinely processed by preschoolers;**

**they are encoded by syntactic means in sentences with distributive quantification**

# Three distributive patterns in Hungarian:

(1)a. ***Mind-három gyerek két autóval játszik***

all three kid two car-with plays

'Every one of three kids are playing with two cars.'

b. ***Három gyerek is két autóval játszik***

three kid DIST two car-with plays

'Three kids each are playing with two cars',

c. ***Három gyerek két-két autóval játszik***

three kid two-two car-with plays

'Three kids are playing with two cars apiece'.

# Psychological background

Lots of evidence of intuitive addition and subtraction since infancy (Wynn 1992, McCrink & Wynn 2004, Barth, La Mont, Lipton, & Spelke 2005, etc.)

**Any evidence of intuitive multiplication???**

6-month old infants notice a change of ratios.

Illiterate fishermen can calculate optimal ratios.

# Inconclusive evidence of whether preschoolers can multiply or do multiple addition

Barth, Baron, Spelke and Carey (2009): kindergarteners are capable of halving, but results are inconclusive as regards doubling.

McCrink & Spelke's (2010): 5-7-year-old children can carry out scalar transformation (doubling, quadrupling, or increasing by 2.5) above chance level.

# Linguistic backgrund

The distributive interpretations of doubly quantified sentences involve multiplication:

(2) ***Three kids are playing with two cars***.

a. ’There are three kids, each of whom is playing with two (possibly different) cars.’

b. ’There are two cars, each of which three (possibly different) kids are playing with.’

c. ’There are three kids and two cars, and the former are are playing with the latter.’

# Languages have means to enforce the distributive readings

**Marking the distributive key** (= the multiplier):

(3)a. ***Mind-három néni*** ***két kutyát*** ***sétáltat.***

all three woman two dog-ACC walks

’Every one of three women is walking two dogs.’

b. ***Három néni*** ***is*** ***két kutyát*** ***sétáltat.***

threewoman DIST two dog-ACC walks

’Three women each are walking two dogs.’

# Languages have means to enforce the distributive readings

**Marking the distributed share** (= the multiplicand):

(4) ***Három néni*** ***két-két*** ***kutyát*** ***sétáltat.***

three woman two-two dog-ACC walks

'Three women are walking two dogs apiece.'

# Distributive scope in child language

**Former experiments**: testing **passive knowledge**, and mostly **multiplication by 1** (Brooks & Braine 1996; Pagliarini et al. 2012; Syrett & Musolino 2013)

(5) ***All of the men/Three men are building a boat.***

Musolino (2009):

(6**)** ***Two boys are holding three balloons.***

É. Kiss, Gerőcs & Zétényi (2013):

(7) ***Két fiú is három autóval játszik***.

two boy DIST three car-with plays

# The experiment

**Participants**:

101 children, 3 age groups:

31 small kids: 4;3–5;5 mean age 4;10

32 big kids: 5;7–6;9 mean age 6;2

38 1st graders: 6;5–7;6 mean age 7;1

# **Materials and methods**:

A warm-up truth-value judgement task:

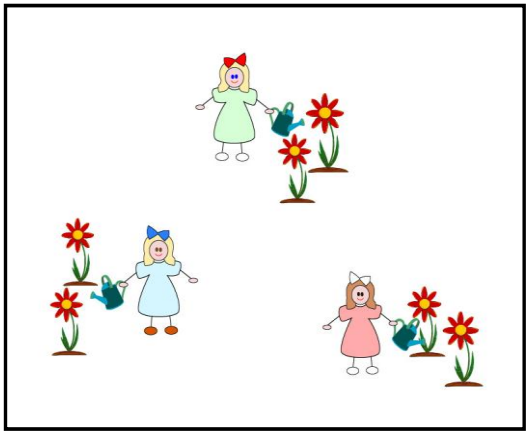
(8)a. ***Mind-három lány két virágot locsol***

every-three girl two car-with plays

’Every one of three girls is playing with two cars.’

b. ***Három lány is két virágot locsol***

c. ***Három lány két-két virágot locsol***



# Test task: Act out with toys

(9)a. ***Mind*** ***a három maci*** ***két cukorkát*** ***kapott***.

all the three bear two candy-ACC got

'Every one of the three bears got two candies.'

Experimenter:

Act this out with these toys!

Here are 3 bears. **How many candies do you need**?

# Test sentences:

b. ***Mind-két embernek*** ***három malac-a*** ***van***.

all-two man-DAT three pig-POSS.3SG is

'Both men have three pigs.'

(10)a. ***Két fánál*** ***is*** ***három bárány álldogál***.

two tree-at DIST three lamb stands

'At each of two trees, three bears are standing.'

b. ***Három néni*** ***is*** ***két kutyá-t*** ***sétáltat***.

three woman DIST two dog-ACC walks

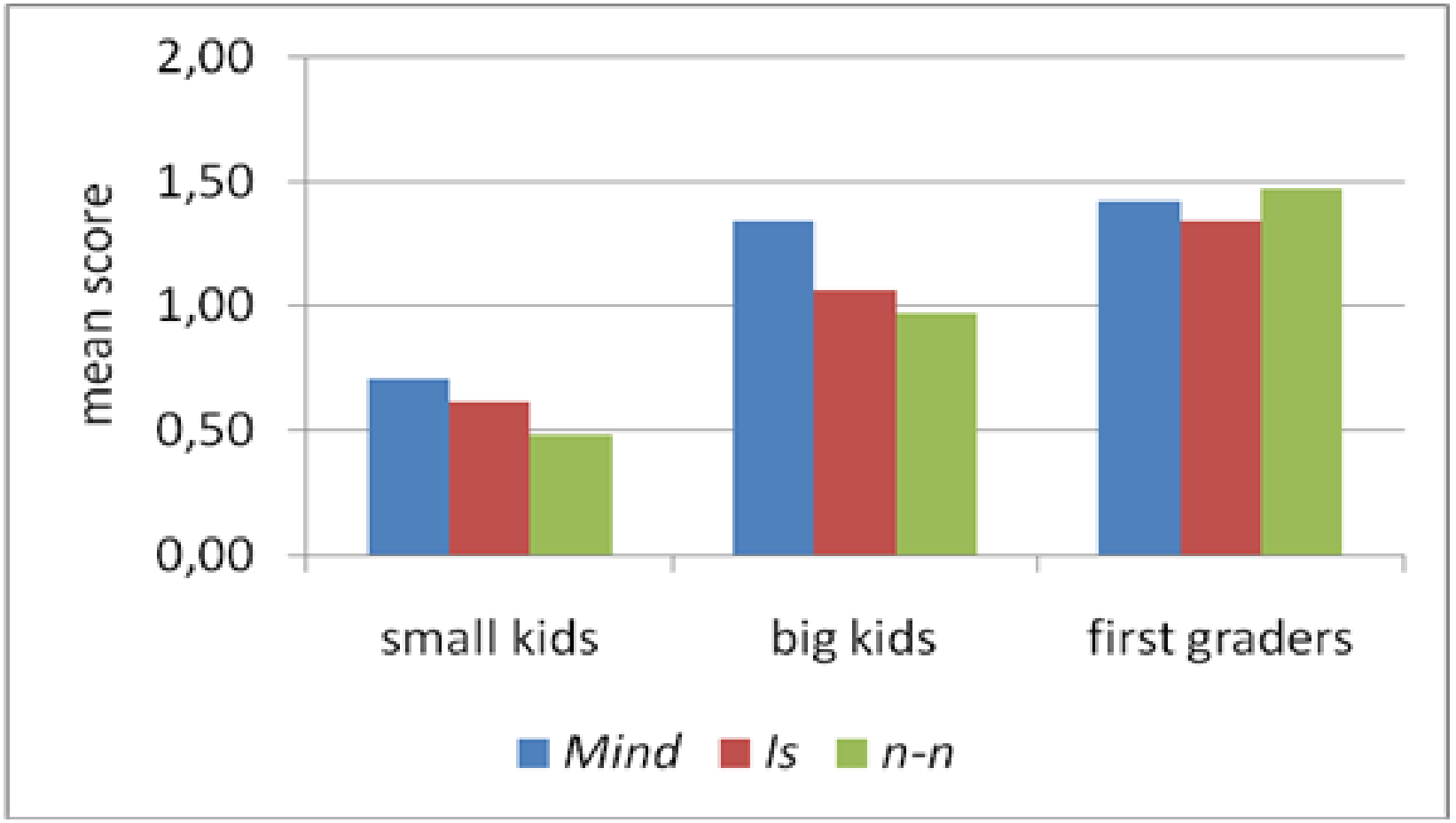
'Three women each are walking two dogs.'

# Test sentences:

(11)a. ***Két autó-t*** ***négy-négy*** ***maci*** ***tol***.
two car-ACC four-four bear pushes
'Two cars are being pushed by four bears apiece.'

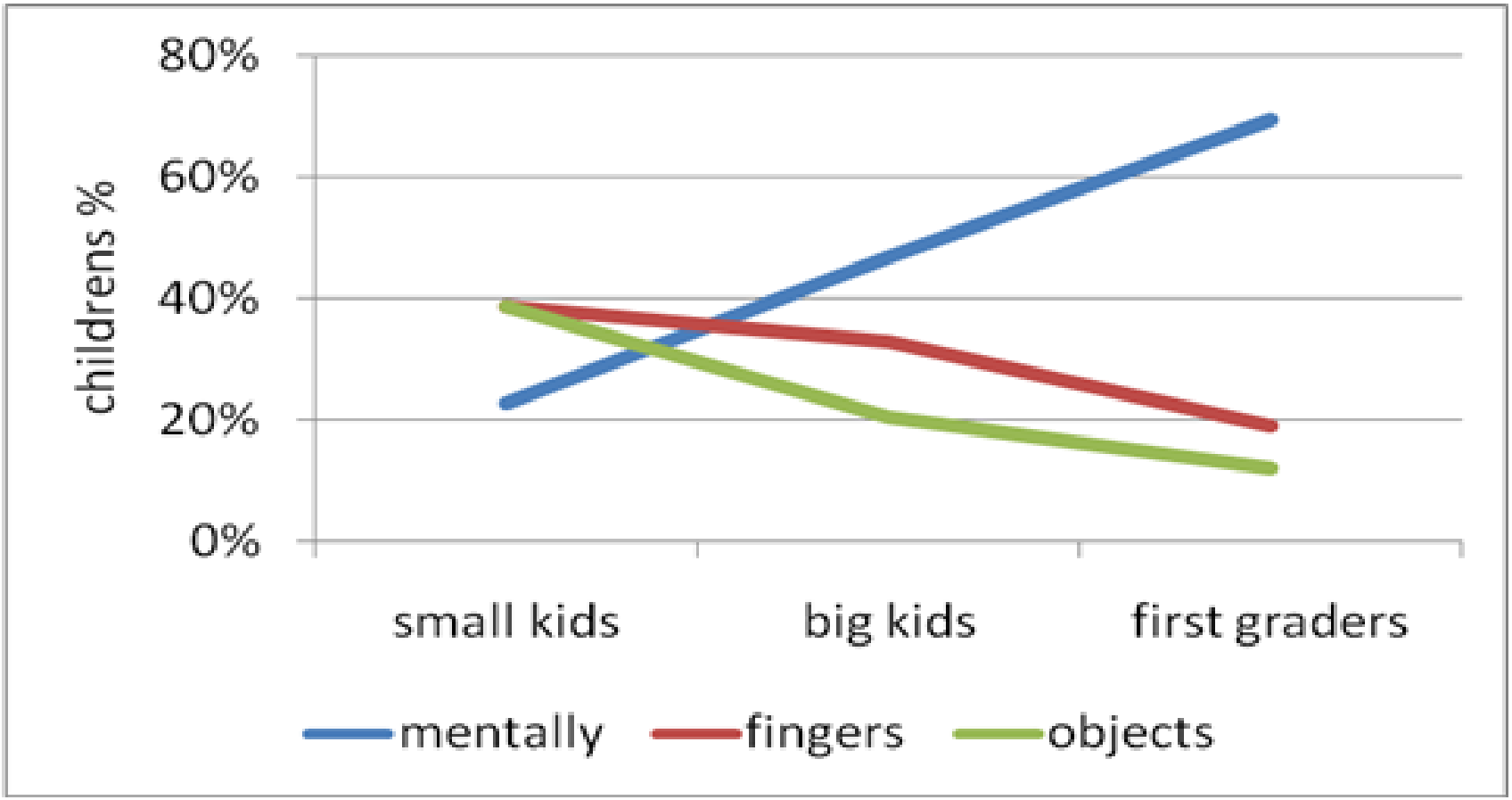
b. ***Három kutya*** ***két-két*** ***bárányra*** ***vigyáz***.
three dog two-two lamb-SUBLAT gards
'Three dogs are shepherding two lambs apiece.'

# Results:
# Mean scores for the 3 types of distributive sentences
(significant (**) growth by age for each)

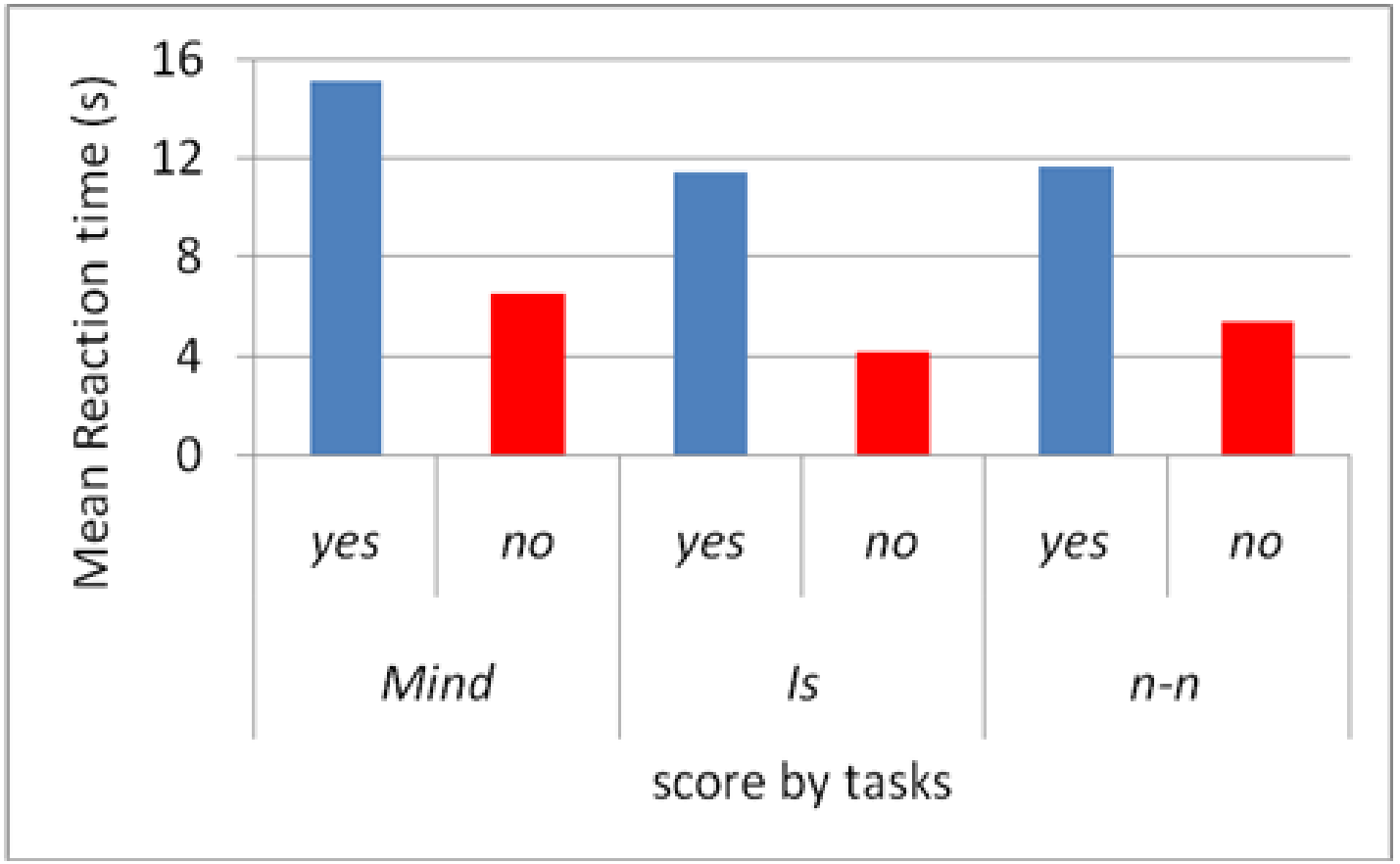


# The mean ages of children with 0, 1, & 2 scores (significant (**) differences for each sentence-type)

# Three strategies of calculating the product of multiplication



# Reaction times of answers achieving 1 or 2 scores and answers achieving no score



# Fillers: multiplication by 1

(12)a. ***Mind az öt bácsi-nak van (egy) autó-ja***.
all the five man-DAT is a car-POSS.3SG
'Every one of the five men has a car.'

b. ***Két bárányt is kerget egy kutya***.
two lamb-ACC DIST chases a dog
'Two lambs each are being chased by a dog.'

c. ***Négy gyerek kapott egy-egy cukorkát***.
four child got one-one candy-ACC
'Four children got one candy a piece.'

# Fillers

Incorrect answers:

| | |
|---|---|
| Small kids: | 13% |
| Big kids: | 11% |
| 1st graders: | 0% |

# Discussion

**Children are capable of calculating the result of multiplication** encoded by a doubly quantified sentence.

Success rate: **30%** at the age of **5**;

over **50%** at the age of **6**,

over **70%** at the age of **7**,

→the ability to carry out multiplication with exact numbers becomes established between 5-7 years.

# The acquisition path of multiplication

**The acquisition of distributivity markers:**
***mind*** 'all' **>** ***is*** **> reduplication**

***mind***: lexically transparent,
most frequent
***is***: a clitic
semantically ambiguous
**numeral redupliation**:
misleading iconicity: it suggests duplication instead of multiplication;
rare: 3000 reduplication vs. 64 000 *mind* in a corpus from 1950-2000

# The acquisition path of multiplication

i. **Multiplication of an individual** (quantifier phrase + indefinite)
ii. **Multiplication of a set** (two quantifier phrases)

Increasing degree of abstractness:

1. multiplying sets of objects + counting them
2. multiplying sets of fingers
3. multiplying mental sets

# Conclusion

The distributive interpretation of doubly quantified sentences involves multiplication.

This is part of the grammar of 6-7-year-old children; i.e., children can perform multiplication prior to arithmetic training.

Language plays a crucial role in numerical cognition; children learn the algorhythm of multiplication as part of language acquisiton.